PROBLEMS

EX 1

A steel rod 25 mm diameter and 4 m long is subjected to an axial pull of 45kN. Find: a) Stress b) Strain c) Elongation. Take $E = 2.1\times10^5\ N/mm^2$

Solution:

Given: $d = 25mm,\ l = 4m = 4000mm,\ P = 45kN = 45000N$

Area, $A = \frac{\pi d^2}{4} = \frac{\pi\times25^2}{4} = 490.87mm^2$; $\therefore$ Stress, $\sigma = \frac{P}{A} = \frac{45000}{490.87} = 91.6N/mm^2$

Elastic modulus, $E = \frac{\sigma}{\varepsilon}$; $\therefore$ Strain, $\varepsilon = \frac{\sigma}{E} = \frac{91.6}{2.1\times10^5} = 4.36\times10^{-4}$

Also, Strain, $\varepsilon = \frac{\delta l}{l}$; $\therefore$ Elongation, $\delta l = \varepsilon\times l = 4.36\times10^{-4}\times4000 = 1.744mm$

EX 2

A steel rod 20mm diameter is subjected to a tensile load of 40kN the extension of rod was found to be 0.5mm in 400mm length. Find: (i) Stress (ii) Strain (iii) Modulus of elasticity

Solution:

Given: $d = 20mm,\ l = 400mm,\ P = 40kN = 40\times10^3 N,\ \delta l = 0.5mm$

Area, $A = \frac{\pi d^2}{4} = \frac{\pi\times20^2}{4} = 314.2mm^2$; $\therefore$ Stress, $\sigma = \frac{P}{A} = \frac{40\times10^3}{314.2} = 127.30N/mm^2$

Strain, $\varepsilon = \frac{\delta l}{l} = \frac{0.5}{400} = 0.00125$; $\therefore$ Elastic modulus, $E = \frac{\sigma}{\varepsilon} = \frac{127.30}{0.00125} = 101840N/mm^2$

P 1

A load of 5 kN is to be raised with the help of a steel wire. Find the diameter of the steel wire, if the stress is not to exceed 100 MPa.

Data : Load, $P = 5\text{ kN} = 5 \times 10^3\text{ N} = 5000\text{ N}$ $(\because 1\text{ kN} = 10^3\text{ N})$

Stress, $\sigma = 100\text{ MPa} = 100 \times 10^6\text{ N/m}^2 = 100\text{ N/mm}^2$ $(\because 1\text{ N/mm}^2 = 10^6\text{ N/m}^2)$

Solution : Let, d = Diameter of steel wire in mm

$\therefore$ Area of cross-section of steel wire, $A = \frac{\pi}{4} d^2$, mm²

$$\text{Allowable or Working stress, } \sigma = \frac{\text{Load}}{\text{Area of cross-section of steel wire}} = \frac{P}{A}$$

$$\text{i.e., } 100 = \frac{5000}{\frac{\pi}{4} \times d^2}\text{ ; i.e., } d^2 = \frac{5000 \times 4}{\pi \times 100}$$

$\therefore$ Diameter of steel wire, $d = 7.98\text{ mm} \simeq 8\text{ mm}$

P 2

A tensile test is performed on a brass specimen 10 mm in diameter using a gauge length of 50 mm. When applying axial tensile load of 25 kN, it was observed that the distance between the gauge marks increase by 0.152 mm, Calculate the Modulus of elasticity of brass.

Data : Diameter of brass specimen, $d = 10$ mm

Gauge length, $l = 50$ mm

Axial tensile load, $P = 25\text{ kN} = 25 \times 1000 = 25000\text{ N}$

Extension of gauge marks, $\delta l = 0.152$ mm

Solution :

$$\text{Area of cross-section of brass specimen, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 10^2 = 78.54\text{ mm}^2$$

$$\text{Stress, } \sigma = \frac{\text{Axial tensile load}}{\text{Area of cross-section of brass-specimen}} = \frac{P}{A} = \frac{25000}{78.54} = 318.31\text{ N/mm}^2$$

$$\text{Strain, } \varepsilon = \frac{\text{Extension of gauge marks}}{\text{Gauge length}} = \frac{\delta l}{l} = \frac{0.152}{50} = 3.04 \times 10^{-3}$$

$$\text{Modulus of elasticity of brass, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{318.31}{3.04 \times 10^{-3}} = 104707.24\text{ N/mm}^2$$

$$= 104.71 \times 10^3\text{ N/mm}^2$$

$\therefore$ Modulus of elasticity of brass, $E = 104.71 \times 10^3\text{ N/mm}^2$.

P 3

A punch with a diameter 20 mm is used to punch a hole in an aluminium plate of thickness 4 mm. If the ultimate shear stress for aluminium is 275 MPa, what force P is required to punch through the plate.

Data : Diameter of punch, $d = 20$ mm

Thickness of plate, $t = 4$ mm

Ultimate shear stress for aluminium, $\tau_u = 275$ MPa $= 275 \times 10^6$ N/m^2

$= 275$ N/mm^2 $\quad (\because 1 \text{ N/mm}^2 = 10^6 \text{ N/m}^2)$

Solution :

$$\text{Sheared area} = \pi d t = \pi \times 20 \times 4 = 251.3274 \text{ mm}^2$$

$$\text{Shearing force, } P = \text{Ultimate shear stress} \times \text{Sheared area}$$

$$= \tau_u \times 251.3274 = 275 \times 251.3274 = 69115.04 \text{ N}$$

$$= 69.115 \times 10^3 \text{ N} = 69.115 \text{ kN} \qquad (\because 10^3 \text{ N} = 1 \text{ kN})$$

P 4

The following data pertains to a tension test conducted in laboratory :

(i) Diameter of the specimen = 15 mm

(ii) Length of specimen = 200 mm

(iii) Extension under a load of 10 kN = 0.035 mm

(iv) Load at yield point = 110 kN

(v) Maximum load = 190 kN

(vi) Length of specimen after failure = 255 mm

(vii) Neck diameter = 12.25 mm

Determine : (i) Young's modulus (ii) Yield stress (iii) Ultimate stress (iv) Percentage elongation (v) Percentage reduction in area (vi) Safe stress adopting factor of safety of 1.5.

Solution :

(i) Young's modulus or Modulus of elasticity

To find Young's modulus, calculate the value of stress and strain within proportionality or elastic limit. As for the load 10 kN which is within proportionality or elastic limit, the extension is given, consider this load.

At a load of 10 kN

$$\text{Stress, } \sigma = \frac{\text{Load}}{\text{Original area of cross-section of specimen}}$$

$$= \frac{P}{A_0} = \frac{10 \text{ kN}}{\frac{\pi}{4} \times d^2} = \frac{10 \times 10^3}{\frac{\pi}{4} \times 15^2} = 56.5884 \text{ N/mm}^2 \quad [\because 1 \text{ kN} = 10^3 \text{ N}]$$

$$\text{Strain corresponding to this load, } \varepsilon = \frac{\text{Increase in length or Extension}}{\text{Original length or Gauge length}}$$

$$= \frac{\delta l}{l} = \frac{0.035}{200} = 1.75 \times 10^{-4}$$

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{56.5884}{1.75 \times 10^{-4}} = 323362.286 \text{ N/mm}^2$$

$$= 323.362 \times 10^3 \text{ N/mm}^2$$

(ii) Yield stress (σ_y)

$$\text{Yield stress} = \frac{\text{Load at yield point}}{\text{Original area of cross-section of specimen}}$$

$$= \frac{110 \text{ kN}}{\frac{\pi}{4} \times d^2} = \frac{110 \times 10^3}{\frac{\pi}{4} \times 15^2} = 622.473 \text{ N/mm}^2$$

(iii) Ultimate stress (σ_u)

$$\text{Ultimate stress} = \frac{\text{Maximum load or Load at ultimate point}}{\text{Original area of cross-section of specimen}}$$

$$= \frac{190 \text{ kN}}{\frac{\pi}{4} \times d^2} = \frac{190 \times 10^3}{\frac{\pi}{4} \times 15^2} = 1075.18 \text{ N/mm}^2$$

(iv) Percentage elongation

$$\text{Percentage elongation} = \frac{\text{Toal increase in length}}{\text{Original length or Gauge length}} \times 100$$

$$= \frac{\text{Length of specimen after failure} - \text{Length of specimen}}{\text{Length of specimen}} \times 100$$

$$= \frac{255 - 200}{200} \times 100 = 27.5\%$$

(v) Percentage reduction in area

$$\text{Percentage reduction in area} = \frac{\text{Original cross-sectional area} - \text{Final cross-sectional area}}{\text{Original cross-sectional area}} \times 100$$

$$= \frac{\text{Original cross-sectional area} - \text{Cross-sectional area at failure}}{\text{Original cross-sectional area}} \times 100$$

$$= \frac{\frac{\pi}{4} \times 15^2 - \frac{\pi}{4} \times 12.25^2}{\frac{\pi}{4} \times 15^2} \times 100 = \frac{15^2 - 12.25^2}{15^2} \times 100 = 33.3\%$$

$$\left[\because \text{Cross-sectional area at failure} = \frac{\pi}{4} \times (\text{Diameter at neck or failure})^2\right]$$

(vi) Safe stress

$$\text{Safe stress} = \frac{\text{Yield stress}}{\text{Factor of safety}} = \frac{622.473}{1.5} = 414.982 \text{ N/mm}^2 \simeq 415 \text{ N/mm}^2$$

P 5

A rod of diameter 15 mm and 50 mm long is subjected to tensile load of 25 kN. The modulus of elasticity for steel rod may be taken as 200 kN/mm². Find stress, strain and elongation of the bar due to applied load.

Data : Diameter of rod, $d = 15$ mm ; Length of rod, $l = 50$ mm

Applied tensile load, $P = 25 \text{ kN} = 25 \times 10^3 \text{ N}$ ($\because 1 \text{ kN} = 10^3 \text{ N}$)

Modulus of elasticity, $E = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$

Solution :

(i) Stress (σ)

$$\text{Stress, } \sigma = \frac{\text{Applied load}}{\text{Area of cross-section of rod}} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{25 \times 10^3}{\frac{\pi}{4} \times 15^2} = 141.471 \text{ N/mm}^2 \text{ (Tensile)}$$

(ii) Strain (ε)

$$\text{Modulus of elasticity, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

$$\text{i.e., } 200 \times 10^3 = \frac{141.471}{\varepsilon}$$

$$\therefore \text{Strain, } \varepsilon = 7.07355 \times 10^{-4}$$

(iii) Elongation (δl)

$$\text{Strain, } \varepsilon = \frac{\text{Change in length or Elongation}}{\text{Length of rod}} = \frac{\delta l}{l}$$

$$\text{i.e., } 7.07355 \times 10^{-4} = \frac{\delta l}{50}$$

$$\therefore \text{Elongation, } \delta l = 0.0354 \text{ mm}$$

P 6

Example : 1.22

A rod of cross-sectional area 15 mm × 15 mm and 1 m long is subjected to a compressive load 22.5 kN. Calculate the stress and decrease in length, if Young's modulus is 200 GN/m².

Data : Cross-sectional area of rod, $A = 15\text{mm} \times 15\text{ mm} = 225\text{ mm}^2$

Length of rod, $l = 1\text{ m} = 1 \times 10^3\text{ mm} = 1000\text{ mm}$ $(\because 1\text{ m} = 10^3\text{ mm})$

Applied compressive load, $P = 22.5\text{ kN} = 22.5 \times 10^3\text{ N}$ $(\because 1\text{ kN} = 10^3\text{ N})$

Young's modulus $E = 200\text{ GN/M}^2 = 200 \times 10^9\text{ N/m}^2 = 200 \times 10^3\text{ N/mm}^2$

$[\because 1\text{ GN} = 10^9\text{ N/mm}^2;\ 10^6\text{ N/m}^2 = 1\text{ N/mm}^2]$

Solution :

(i) Stress (σ)

$$\text{Stress, } \sigma = \frac{\text{Applied compressive load}}{\text{Area of cross-section of rod}} = \frac{P}{A} = \frac{22.5 \times 10^3}{225} = 100\text{ N/mm}^2\text{ (Compressive)}$$

(ii) Decrease in length (δl)

$$\text{Modulus of elasticity or Young's modulus, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

$$\text{i.e., } 200 \times 10^3 = \frac{100}{\varepsilon}$$

$$\therefore \text{ Strain, } \varepsilon = 5 \times 10^{-4}$$

$$\text{Also, Strain, } \varepsilon = \frac{\text{Change in length or Decrease in length}}{\text{Length of rod}} = \frac{\delta l}{l}$$

$$\text{i.e., } 5 \times 10^{-4} = \frac{\delta l}{1000}$$

$$\therefore \text{ Decrease in length, } \delta l = 0.5\text{ mm}$$

P 7

A load of 4 kN is to be raised with the help of a steel wire. The permissible tensile stress should not exceed 70 N/mm². What is the minimum diameter of wire required? What will be extension for 3.5 m length of wire? Assume Young's modulus is 196.2 GN/m².

Data : Load, $P = 4\ kN = 4 \times 10^3\ N = 4000\ N$ ($\because 1\ kN = 10^3\ N$)

Permissible tensile stress, $\sigma = 70\ N/mm^2$

Length of steel wire, $l = 3.5\ m = 3.5 \times 10^3\ mm = 3500\ mm$ ($\because 1\ m = 10^3\ mm$)

Young's modulus, $E = 196.2\ GN/m^2 = 196.2 \times 10^9\ N/m^2 = 196.2 \times 10^3\ N/mm^2$

($\because 1\ GN = 10^9\ N/m^2$; $10^6\ N/m^2 = 1\ N/mm^2$)

Solution : (i) Minimum diameter of steel wire

Let, d = Diameter of steel wire in mm

$\therefore$ Area of cross-section of steel wire, $A = \frac{\pi}{4} \times d^2\ mm^2$.

$$\text{Permissible tensile stress, } \sigma = \frac{\text{Load}}{\text{Area of cross-section of steel wire}} = \frac{P}{A}$$

$$\text{i.e., } 70 = \frac{4000}{\frac{\pi}{4} \times d^2}\ ;\ \text{i.e., } d^2 = \frac{4000 \times 4}{\pi \times 70}$$

$\therefore$ Minimum diameter of steel wire, $d = 8.53$ mm

(ii) Extension of steel wire

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

$$\text{i.e., } 196.2 \times 10^3 = \frac{70}{\varepsilon}$$

$$\therefore \text{Strain, } \varepsilon = 3.5678 \times 10^{-4}$$

$$\text{Also, Strain, } \varepsilon = \frac{\text{Change in length or Extension}}{\text{Length of steel wire}} = \frac{\delta l}{l}$$

$$\text{i.e., } 3.5678 \times 10^{-4} = \frac{\delta l}{3500}$$

$\therefore$ Extension of steel wire, $\delta l = 1.2487\ mm \simeq 1.25\ mm$

P 8

A mild steel bar of 15 mm diameter was subjected to tensile test. The test bar was found to yield a load of 90 kN, attains a maximum load of 180 kN and ultimately fails at a load of 67.5 kN. Determine the following :

Tensile stress at yield point, ultimate stress and stress at breaking point, if the diameter of the neck is 7.5 mm.

Data : Diameter of steel bar, $d = 15$ mm

Load at yield point = 90 kN = 90×10^3 N ($\because$ 1 kN = 10^3 N)

Maximum load = 180 kN = 180×10^3 N

Load at breaking point = 67.5 kN = 67.5×10^3 N

Neck diameter = 7.5 mm

Solution :

(i) Tensile stress at yield point (σ_y)

$$\text{Tensile stress at yield point, } \sigma_y = \frac{\text{Load at yield point}}{\text{Original area of cross-section of bar}}$$

$$= \frac{90 \text{ kN}}{\frac{\pi}{4} \times d^2} = \frac{90 \times 10^3}{\frac{\pi}{4} \times 15^2} = 509.3 \text{ N/mm}^2$$

(ii) Ultimate stress (σ_u)

$$\text{Ultimate stress, } \sigma_u = \frac{\text{Maximum load}}{\text{Original area of cross-section of bar}}$$

$$= \frac{180 \text{ kN}}{\frac{\pi}{4} \times d^2} = \frac{180 \times 10^3}{\frac{\pi}{4} \times 15^2} = 1018.6 \text{ N/mm}^2$$

(iii) Stress at breaking point (σ_B)

$$\text{Nominal stress at breaking point, } \sigma_B = \frac{\text{Load at breaking point}}{\text{Original area of cross-section of bar}}$$

$$= \frac{67.5 \text{ kN}}{\frac{\pi}{4} \times d^2} = \frac{67.5 \times 10^3}{\frac{\pi}{4} \times 15^2} = 381.972 \text{ N/mm}^2$$

P 9

The following data pertains to a tension test conducted in laboratory :

(i) Diameter of specimen = 20 mm

(ii) Gauge length of specimen = 100 mm

(iii) Final length = 130 mm

(iv) Final diameter = 11.5 mm

(v) Yield load = 92 kN

(vi) Ultimate load = 165 kN

Determine : (i) Yield stress (ii) Ultimate tensile stress (iii) Percentage elongation (i Percentage reduction in area.

Solution :

(i) Yield stress (σ_y)

$$\text{Yield stress, } \sigma_y = \frac{\text{Load at yield point}}{\text{Original area of cross-section of specimen}}$$

$$= \frac{92 \text{ kN}}{\frac{\pi}{4} \times d^2} = \frac{92 \times 10^3}{\frac{\pi}{4} \times 20^2} = 292.845 \text{ N/mm}^2 \qquad [\because 1 \text{ kN} = 10^3 \text{ N}]$$

(ii) Ultimate tensile stress (σ_u)

$$\text{Ultimate tensile stress, } \sigma_u = \frac{\text{Ultimate load}}{\text{Original area of cross-section of specimen}}$$

$$= \frac{165 \text{ kN}}{\frac{\pi}{4} \times d^2} = \frac{165 \times 10^3}{\frac{\pi}{4} \times 20^2} = 525.21 \text{ N/mm}^2 \qquad [\because 1 \text{ kN} = 10^3 \text{ N}]$$

(iii) Percentage elongation

$$\text{Percentage elongation} = \frac{\text{Total increase in length of specimen}}{\text{Gauge length of specimen}} \times 100$$

$$= \frac{\text{Final length} - \text{Gauge length}}{\text{Gauge length}} \times 100 = \frac{130 - 100}{100} \times 100 = 30\%$$

(iv) Percentage reduction in area

$$\text{Percentage reduction in area} = \frac{\text{Original cross-sectional area} - \text{Final cross-sectional area}}{\text{Original cross-sectional area}} \times 100$$

$$= \frac{\frac{\pi}{4} \times 20^2 - \frac{\pi}{4} \times 11.5^2}{\frac{\pi}{4} \times 20^2} \times 100 = \frac{20^2 - 11.5^2}{20^2} \times 100$$

$$= 66.947\ \%$$

$$[\because \text{Final cross-sectional area} = \frac{\pi}{4} \times (\text{Final diameter})^2]$$

P 10

A hollow steel column has to carry an axial load of 3 MN. If the external diameter of the column is 300 mm, find the internal diameter. The ultimate stress for steel is to be 480 N/mm². Take factor of safety as 4.

Data : Axial load on the column, $P = 3\ \text{MN} = 3 \times 10^6\ \text{N}$ ($\because 1\ \text{MN} = 10^6\ \text{N}$)

External diameter, $d_o = 300$ mm

Ultimate stress for steel, $\sigma_u = 480\ \text{N/mm}^2$

Factor of safety = 4

Let, Internal diameter $= d_i$

$$\text{Safe or allowable stress, } \sigma = \frac{\text{Ultimate stress}}{\text{Factor of safety}} = \frac{480}{4} = 120\ \text{N/mm}^2$$

$$\text{Cross-sectional area of steel column, } A = \frac{\pi}{4} \times \left(d_o^2 - d_i^2\right) = \frac{\pi}{4} \times \left(300^2 - d_i^2\right)$$

$$\text{Safe stress, } \sigma = \frac{\text{Axial load}}{\text{Cross-sectional area of column}} = \frac{P}{\frac{\pi}{4} \times \left(d_o^2 - d_i^2\right)}$$

$$\text{i.e., } 120 = \frac{3 \times 10^6}{\frac{\pi}{4} \times \left(300^2 - d_i^2\right)}\ ; \quad \text{i.e, } \left(300^2 - d_i^2\right) = \frac{3 \times 10^6 \times 4}{120 \times \pi}$$

$$\text{i.e., } 90000 - d_i^2 = 31831\ ; \qquad \text{i.e., } d_i^2 = (90000 - 31831)$$

$\therefore$ Internal diameter of steel column, $d_i = \sqrt{58169} = 241.2$ mm

P 11

A steel rod 30 mm × 12.5 mm and 500 mm long is subjected to an axial pull of 75 kN. Determine the changes in length, width, thickness and volume of bar. Young's modulus is 200 kN/mm² and Poisson's ratio is 0.3.

Data : Length of rod, $l = 500$ mm ; Width of rod, $b = 30$ mm ; Thickness of rod, $t = 12.5$ mm,

Axial pull, $P = 75$ kN $= 75 \times 10^3$ N,

Young's modulus, $E = 200$ kN/mm² $= 200 \times 10^3$ N/mm² ($\because$ 1 kN $= 10^3$ N

Poisson's ratio, $\frac{1}{m} = 0.3$

Solution :

(i) Change in length

Cross-sectional area of the rod = Width or rod × Thickness of rod

i.e., $A = b \times t = 30 \times 12.5 = 375$ mm²

$$\text{Stress in the direction of load, } \sigma = \frac{\text{Axial pull}}{\text{Cross-sectional area of rod}}$$

$$= \frac{P}{b \times t} = \frac{75 \times 10^3}{375} = 200 \text{ N/mm}^2$$

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Longitudinal or Linear strain}} = \frac{\sigma}{\varepsilon}$$

$$\therefore \text{Longitudinal or Linear strain, } \varepsilon = \frac{\sigma}{E} = \frac{200}{200 \times 10^3} = 10^{-3}$$

$$\text{Also, Longitudinal strain, } \varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l}$$

$\therefore$ Change in length, $\delta l = \varepsilon \times l = 10^{-3} \times 500 = 0.$ [illegible] mm

(ii) Change in width

$$\text{Poisson's ratio, } \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\therefore \text{Laternal strain} = \frac{1}{m} \times \text{Longitudinal strain} = 0.3 \times 10^{-3} = 3 \times 10^{-4}$$

$$\text{Also, Lateral strain} = \frac{\text{Change in width}}{\text{Original width}} = \frac{\text{Change in thickness}}{\text{Original thickness}} = \frac{\delta b}{b} = \frac{\delta t}{t}$$

$\therefore$ Change in width, δb = Lateral strain × Original width (b) $= 3 \times 10^{-4} \times 30 = 9 \times 10^{-3}$ mm

(iii) Change in thickness

Change in thickness δt = Lateral strain × Original thickness (t)

$= 3 \times 10^{-4} \times 12.5 = 3.75 \times 10^{-3}$ mm

(iv) Change in volume

Original volume, $v = lbt = 500 \times 30 \times 125 = 187500$ mm³

Let, Change in volume $= \delta v$

$$\text{Volumetric strain, } \varepsilon_v = \frac{\delta v}{v} = \frac{\delta l}{l}\left(1 - \frac{2}{m}\right) = \text{Longitudinal strain} \times \left(1 - \frac{2}{m}\right)$$

$$\text{i.e., } \frac{\delta v}{187500} = 10^{-3} \times (1 - 2 \times 0.3)$$

$\therefore$ Change in volume, $\delta v = 75$ mm³ [As the value of δv is +ve, it is increase in volum

P 12

A steel bar 2.4 m long and 30 mm square is elongated by a load 400 kN. If Poisson's ratio is 0.25, find the increase in volume. Assume E = 200 kN/mm².

Data : Length of bar, $l = 2.4$ m $= 2.4 \times 10^3 = 2400$ mm ($\because$ 1 m = 10^3 mm)

Width of bar, $b = 30$ mm

Thickness of bar, $t = 30$ mm ($\because$ Square cross-section)

Poisson's ratio, $\frac{1}{m} = 0.25$; Load $P = 400$ kN $= 400 \times 10^{-3}$ N

Young's modulus, $E = 200$ kN/mm² $= 200 \times 10^3$ N/mm² ($\because$ 1 kN = 10^3 N)

Solution :

$$\text{Stress in the direction of load, } \sigma = \frac{\text{Axial load}}{\text{Cross-sectional area of bar}} = \frac{P}{b \times t} = \frac{400 \times 10^3}{30 \times 30}$$

$\therefore$ Stress, $\sigma = 444.444$ N/mm²

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Linear strain}} = \frac{\sigma}{\varepsilon}$$

$$\therefore \text{ Linear strain, } \varepsilon = \frac{\sigma}{E} = \frac{444.444}{200 \times 10^3} = 2.222 \times 10^{-3}$$

Original volume, $v = lbt = 2400 \times 30 \times 30 = 2160000$ mm³ $= 2.16 \times 10^6$ mm³

Let, Change in volume $= \delta v$

$$\text{Volumetric strain, } \varepsilon_v = \frac{\delta v}{v} = \frac{\delta l}{l} \times \left(1 - \frac{2}{m}\right) = \text{linear strain}\left(1 - \frac{2}{m}\right)$$

$$\text{i.e., } \frac{\delta v}{2.16 \times 10^6} = 2.222 \times 10^{-3}(1 - 2 \times 0.25)$$

$\therefore$ Increase in volume, $\delta v = 2400$ mm² [As δv is +ve, it is increase in volume]

P 13

The Young's modulus for a given material is 100 kN/mm² and its Modulus of rigidity 40 kN/mm². Determine its Bulk modulus and also its lateral contraction, if the diamel is 50 mm, length 2 m and extension 2 mm.

Data : Young's modulus, $E = 100$ kN/mm² $= 100 \times 10^3$ N/mm² ($\because$ 1 kN = 10^3 N)

Modulus of rigidity, G or $C = 40$ kN/mm² $= 40 \times 10^3$ N/mm²

Diameter of bar, $d = 50$ mm ; Length of bar, $l = 2$ m $= 2 \times 10^3$ mm $= 2000$ mm

Extension of bar, $\delta l = 2$ mm

Solution :

(i) Bulk modulus

Reaction between Young's modulus, Modulus of rigidity and Bulk modulus is

$$\text{Young's modulus, } E = \frac{9KG}{G+3K} \text{ or } \frac{9KC}{C+3K} \text{ where } K = \text{Bulk modulus}$$

$$\text{i.e., } 100 \times 10^3 = \frac{9K \times 40 \times 10^3}{(40 \times 10^3 + 3K)}$$

$$\text{i.e., } (40 \times 10^3 + 3K) = \frac{(9K)(40 \times 10^3)}{(100 \times 10^3)} = 3.6\text{ K}$$

$$\text{i.e., } 40 \times 10^3 = 3.6\text{ K} - 3\text{ K} = K(3.6 - 3) = 0.6\text{ K}$$

$$\therefore \text{ Bulk modulus, } K = \frac{40 \times 10^3}{0.6} = 66.667 \times 10^3 \text{ N/mm}^2$$

(ii) Lateral contraction

$$\text{Poisson's ratio, } \frac{1}{m} = \frac{3K - 2C}{6K + 2C} = \frac{3 \times 66.667 \times 10^3 - 2 \times 40 \times 10^3}{6 \times 66.667 \times 10^3 + 2 \times 40 \times 10^3} = 0.25$$

OR

$$\text{Modulus of rigidity, } C = \frac{mE}{2(m+1)} \text{ where } E = \text{Young's modulus}$$

$$\text{i.e., } 40 \times 10^3 = \frac{m \times 100 \times 10^3}{2(m+1)}$$

$$\text{i.e., } m + 1 = \frac{m \times 100 \times 10^3}{2 \times 40 \times 10^3} = 1.25\text{ m}$$

$$\text{i.e., } 1 = 1.25\text{ m} - m = m(1.25 - 1) = 0.25\text{ m}$$

$$\therefore \text{ Poisson's ratio, } \frac{1}{m} = 0.25$$

$$\text{Longitudinal strain, } \varepsilon = \frac{\delta l}{l} = \frac{2}{2000} = 10^{-3}$$

$$\text{Now, Poisson's ratio, } \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\therefore \text{ Lateral strain} = \frac{1}{m} \times \text{Longitudinal strain} = 0.25 \times 10^{-3} = 2.5 \times 10^{-4}$$

$$\text{Also, Lateral strain} = \frac{\text{Change in diameter}}{\text{Original diameter}} = \frac{\delta d}{d}$$

$$\therefore \text{Lateral contraction or change in diameter, } \delta d = \text{Lateral strain} \times d = 2.5 \times 10^{-4} \times 50 = 0.0125\text{ mm}$$

P 14

A bar of steel 1 m long, 50 mm wide and 10 mm thickness is subjected to an axial load of 10 kN in the direction of its length. Find the changes in length, width, thickness and volume. Young's modulus is 200 kN/mm² and Poisson's ratio is 0.25.

Data : Length of bar, $l = 1$ m $= 1 \times 10^3$ mm $= 1000$ mm ($\because$ 1 m $= 10^3$ mm)

Width of bar, $b = 50$ mm ; Thickness of bar, $t = 10$ mm

Young's modulus, $E = 200$ kN/mm² $= 200 \times 10^3$ N/mm² [$\because$ 1 kN $= 10^3$ N]

Poisson's ratio, $\frac{1}{m} = 0.25$; Axial load, $P = 10$ kN $= 10 \times 10^3$ N

Stress in the direction of load, $\sigma = \frac{\text{Axial load}}{\text{Cross-sectional area of bar}} = \frac{P}{b \times t} = \frac{10 \times 10^3}{50 \times 10} = 20$ N/m

$$\text{Young's modulus}, E = \frac{\text{Stress}}{\text{Longitudinal strain}} = \frac{\sigma}{\varepsilon}$$

$$\therefore \text{ Longitudinal strain}, \varepsilon = \frac{\sigma}{E} = \frac{20}{200 \times 10^3} = 10^{-4}$$

$$\text{Also, Longitudinal strain}, \varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l}$$

$$\therefore \text{ Change in length}, \delta l = \varepsilon \times l = 10^{-4} \times 1000 = 0.1 \text{ mm}$$

(ii) Change in width

$$\text{Poisson's ratio}, \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\therefore \text{ Lateral strain} = \frac{1}{m} \times \text{Longitudinal strain} = 0.25 \times 10^{-4} = 2.5 \times 10^{-5}$$

$$\text{Also, Lateral strain} = \frac{\text{Change in width}}{\text{Original width}} = \frac{\text{Change in thickness}}{\text{Original thickness}} = \frac{\delta b}{b} = \frac{\delta t}{t}$$

$$\therefore \text{ Change in width}, \delta b = \text{Laternal strain} \times b = 2.5 \times 10^{-5} \times 50 = 1.25 \times 10^{-3} \text{ mm}$$

(iii) Change in thickness

$$\text{Change in thickness}, \delta t = \text{Lateral strain} \times t = 2.5 \times 10^{-5} \times 10 = 2.5 \times 10^{-4} \text{ mm}$$

(iv) Change in volume

$$\text{Original volume}, v = lbt = 1000 \times 50 \times 10 = 500000 \text{ mm}^3$$

Let, Change in volume $= \delta v$

$$\text{Volumetric strain}, \varepsilon_v = \frac{\delta v}{v} = \frac{\delta l}{l}\left(1 - \frac{2}{m}\right) = \text{Longitudinal strain} \times \left(1 - \frac{2}{m}\right)$$

$$\text{i.e., } \frac{\delta v}{500000} = 10^{-4}(1 - 2 \times 0.25)$$

$\therefore$ Change in volume, $\delta v = 25$ mm³ [As the value of δv is +ve, it is increase in volume

P 15

A bar of 30 mm diameter is subjected to an axial pull of 80 kN. The measured extensi is 0.1 m on a gauge length of 200 mm and the change in diameter is 0.004 mm, Calcul the Poisson's ratio and the values of Young's modulus, Bulk modulus and Modulus rigidity.

Data : Diameter of bar, $d = 30$ mm

Extension of bar, $\delta l = 0.1$ mm ; Gauge length, $l = 200$ mm

Change in diameter, $\delta d = 0.004$ mm

Axial pull or Applied tensile load, $P = 80$ kN $= 80 \times 10^3$ N ($\because$ 1 kN $= 10^3$ N)

Solution : (i) Poisson's ratio $\left(\frac{1}{m}\right)$

$$\text{Linear or Longitudinal strain}, \varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l} = \frac{0.1}{200} = 5 \times 10^{-4}$$

$$\text{Lateral strain} = \frac{\text{Change in diameter}}{\text{Original diameter}} = \frac{\delta d}{d} = \frac{0.004}{30} = 1.333 \times 10^{-4}$$

$$\text{Poisson's ratio}, \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{1.333 \times 10^{-4}}{5 \times 10^{-4}} = 0.2667$$

(ii) Young's modulus

$$\text{Longitudinal stress}, \sigma = \frac{\text{Axial pull}}{\text{Cross-sectional area of bar}} = \frac{P}{A} = \frac{80 \times 10^3}{\frac{\pi}{4} \times d^2}$$

$$= \frac{80 \times 10^3 \times 4}{\pi \times 30^2} = 113.18 \text{ N/mm}^2$$

$$\text{Young's modulus}, E = \frac{\text{Stress}}{\text{Linear strain}} = \frac{\sigma}{\varepsilon} = \frac{113.18}{5 \times 10^{-4}} = 226.36 \times 10^3 \text{ N/mm}^2$$

(iii) Bulk modulus (K)

$$\text{Young's modulus}, E = 3K\left(1 - \frac{2}{m}\right) = 3K\left(\frac{m-2}{m}\right) \quad \text{where } K = \text{Bulk modulus}$$

$$\therefore \quad \text{Bulk modulus}, K = \frac{mE}{3(m-2)} = \frac{3.75 \times 226.36 \times 10^3}{3(3.75 - 2)}$$

$$= 161.686 \times 10^3 \text{ N/mm}^2 \qquad \left[\frac{1}{m} = 0.2667 \; ; \; \therefore \; m = 3.75\right]$$

(iv) Modulus of rigidity or Shear modulus (G or C)

$$\text{Young's modulus}, E = 2C\left(1 + \frac{1}{m}\right) = 2C\left(\frac{m+1}{m}\right) \quad \text{where } C = \text{Modulus of rigidity}$$

$$\therefore \quad \text{Modulus of rigidity}, C = \frac{mE}{2(m+1)} = \frac{3.75 \times 226.36 \times 10^3}{2(3.75 + 1)} = 89.35 \times 10^3 \text{ N/mm}^2$$

END

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