THEODOLITE SURVEYING

Unit 1

1.1 INTRODUCTION

The theodolite is the most accurate instrument used mainly for measuring horizontal and vertical angles. It can also be used for locating points on a line, prolonging survey lines, finding the differences in elevations, setting out grades, ranging curves, etc. Theodolites enable the surveyor to measure angles to an accuracy ranging from 1' to 0.1".

1.2 TYPES OF THEODOLITE

Theodlite primarily classified as

a) Non-Transit theodolite.

b) Transit theodolite

a) Non-Transit theodolite

In case of Non-transit theodolite, the telescope cannot be revolved round the horizontal axis in a vertical plane. These instruments are compact in nature. Such theodolites are obsolete nowadays. Examples are the Y-theodolite and Everest theodolite.

b) Transit theodolite

In case of transit theodolite, the telescope can transit i.e. make complete rotation about its horizontal axis in a vertical plane by 180^0.

All modern theodolites are transit types only, hence transit theodolite name may be replaced by theodolite.

Depending upon the facilities provided for reading of observations the theodolites may be classified as

1. Vernier transit theodolite
2. Optical (glass arc) theodolite
3. Electronic theodolite

All the modern theodolites are of the optical or electronic digital type. A modern theodolite is compact, light in weight and simple in design .All the movable parts and scales are fully enclosed and virtually dust and moisture proof.

The size of a theodolite generally defined by the diameter of the graduated circle on the lower plate and the theodolites are available in sizes ranging from 80mm to 250mm. For example, a 20 cm theodolite means the diameter of the graduated circle of the lower plate is 20 cm.

This chapter mainly focused on the vernier theodolite as it is the basic model for theodolite surveying practice and easy to understand the basic principles of theodolite working.

1. Vernier transit theodolite

This is most commonly used. In this type of theodolites, verniers are provided for reading horizontal and vertical graduated circles. In vernier theodolites angles are read accurately up to 20".

2. Optical theodolite

The precise optical theodolite uses an optical system to read both horizontal and vertical circles. They are more compact and light. The graduations are made on a glass circle and improved micrometers are used. Observer can take readings up to 1" or even less.

3. Electronic theodolites or Digital Theodolite

This type of theodolite angles are displayed in the visual digital form. They are recently developed precise instruments. The readings can be read, recorded, stored, processed and displayed. The precision of this type of instrument varies in the order of 1" to 10".

Electronic theodolites contain circular encoders which sense the rotations of the spindles and the telescope, convert these rotations into horizontal and vertical (or zenith) angles electronically, and display the values of the angles on liquid crystal displays (LCD) or light emitting diode displays (LED).

COMPARISON BETWEEN TRANSIT THEODOLITE AND DUMPY LEVEL

No.	Transit theodolite	Dumpy level
1.	Can be used as a level for small work to take staff readings.	Cannot be used as theodolite to take horizontal and vertical angles.
1.	To make line of sight horizontal, first vertical circle should read 0^0. (i.e., vertical circle should clamped to read zero.)	There is no vertical circle, If level is centred, line of sight will be horizontal.
2.	Permanent adjustment of theodolite involves establishing relationship between five fundamental lines.	Permanent adjustment of dumpy level involves establishing relationship between three fundamental lines.
3.	Because of its variety of uses, such as the measurement of horizontal and vertical angles, prolonging a straight line, measurement of bearings, and measurement of horizontal and vertical distances it is sometimes referred to as universal instrument.	It is mainly used to take staff readings consisting of horizontal line of sight.
4.	Theodolite can be used only for ordinary levelling, since the altitude bubble is not that sensitive as, in the case, of a level	For leveling, dumpy level is better instrument to be used.

4 COMPONENT PARTS OF A TRANIST THEODOLITE

Each type of theodolite is peculiar in its construction and mode of operation. However, inherent ndamentals of all are same. In this course, the details will be considered for vernier type theodolite iich is most popular and is being widely used. Figure 1.1 shows the cross-sectional elevation of ransit theodolite. Following are the various parts of transit theodolite.

1. Levelling Head
2. Shifting Head
3. Lower Plate
4. Upper Plate
5. Plate Levels
6. Standard (or a Frame)
7. Vernier Frame
8. Telescope
9. Vertical Circle
10. Altitude Bubble
11. Screws

12. Spindles
13. Tripod Stand

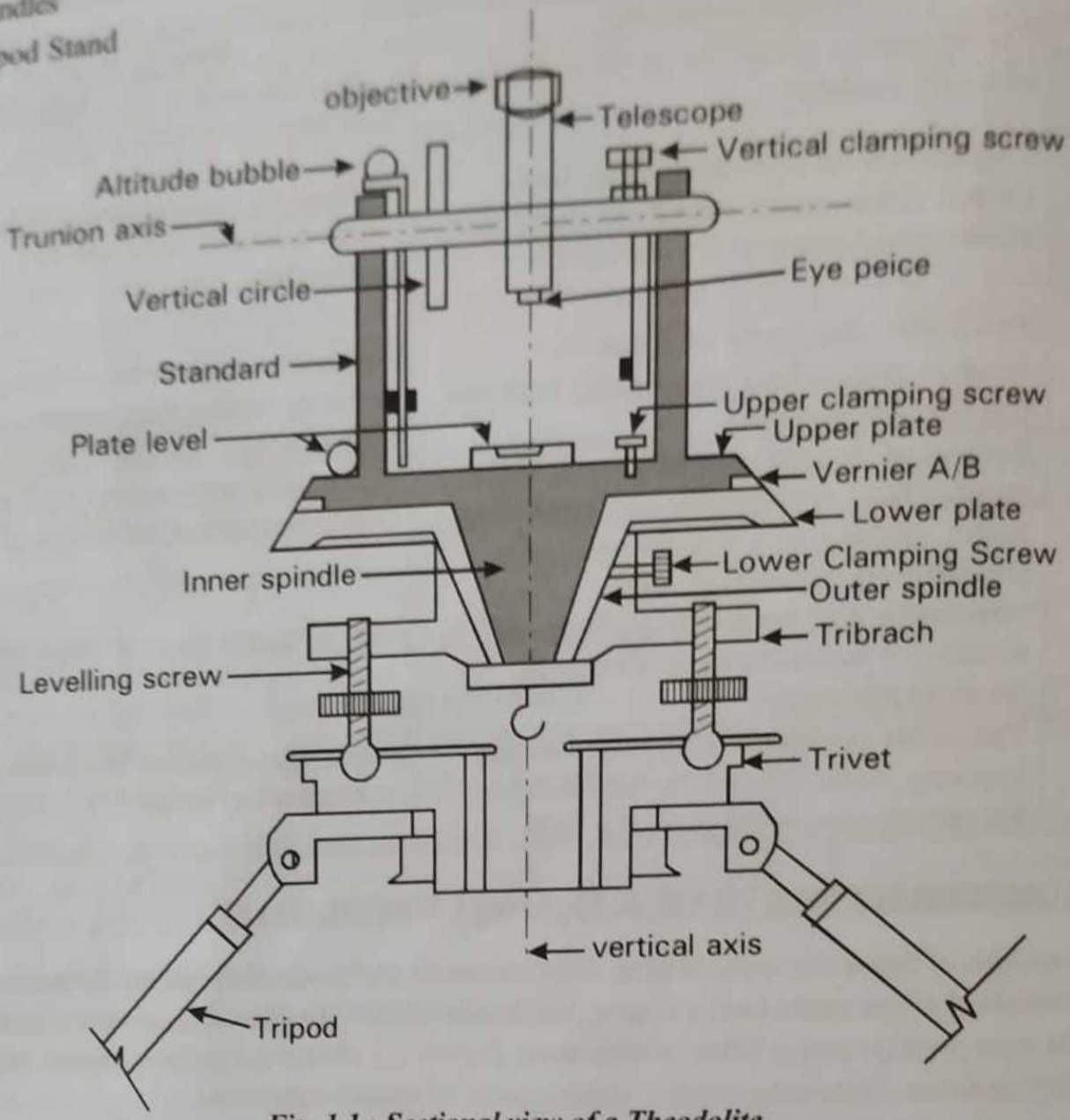


Fig. 1.1 : Sectional view of a Theodolite

Brief description of various parts of a theodolite :

1. Levelling Head

It is the lowermost part of a theodolite. It consists of two parallel horizontal plates separated by three levelling screws.

The lower plate with a large threaded hole in its centre is called trivet or foot plate. It provides a means to place the instrument on (tripod) stand and get it screwed. Its central aperture provides a way for suspending a plumb bob.

The upper plate of the levelling head is called the tribrach. It contains a tapered bearing at the centre. It has three arms each carrying a levelling screw. It provides a support for the upper part of the instrument.

The levelling head supports the entire instrument at its upper part. It also serves to mount the instrument on a tripod stand and for levelling the instrument.

2. Shifting Head

The shifting head is used for exact centring of the instrument after levelling has been completed.

It is a self centering device placed below the lower plate to centre the instrument quickly and accurately. It consists of two plates one sliding over the other by a small area of about 10 mm radius. The two plates can be tightened in the required position. When this device is loosened, the whole instrument can be shifted horizontally within a circle of about 10 mm diameter. The instrument is first approximately centered over the station by moving the tripod legs. Then, exact centering is done by using the shifting head.

3. Lower Plate

It is a graduated horizontal circular plate attached to the outer hollow spindle. It is known as a scale plate or main plate. It is graduated in clockwise direction from 0° to 360° with graduations at 20'. Each fifth degree is numbered. The horizontal circle reading is taken by means of verniers A and B. The least count of the vernier is 20".

The lower plate is provided with the lower clamp screw and a tangent screw. When the lower clamp screw is tightened, the outer hollow spindle is fixed to the tribrach and the lower plate is fixed in position. Finer movement of lower plate is done by turning the lower tangent screw after tightening lower clamp screw.

The diameter of the lower plate (scale plate) varies from 100mm to 150mm. The size of a theodolite is designated by diameter of this plate. Example: 100mm theodolite, 130mm theodolite, etc.

4. Upper Plate (or vernier plate)

Upper plate is attached to the inner solid spindle. It carries two verniers designated as Vernier *A* and Vernier *B* at its two extremities diametrically opposite to each other at 180° apart with magnifiers to magnify the readings. The upper plate supports the standard at its upper surface. The upper plate is provided with the upper clamp screw and a tangent screw for fixing the upper plate to the lower plate. When the upper clamp is clamped, both the upper and lower plates move together as one unit and Vernier readings do not change. When upper clamp is loosened, upper plate rotates with respect to lower plate and vernier readings change.

5. Plate Levels

Pair of level tubes are placed at right angles on the upper plate. The plate bubble is used to make the vertical axis of the instrument truly vertical i.e., for levelling of the instrument.

6. Standard (or A Frame)

Two standards resembling the letter A are attached on the upper plate. These provide the bearings of the pivots of the telescope allowing it to rotate on its trunion axis in vertical plane.

The vernier frame and arm of vertical circle clamp are also attached to it.

7. Vernier Frame

Also called T -frame or index frame, consists of a vertical leg known as clipping arm and a horizontal bar called the index arm engraved with verniers C and D at its ends. Each of the verniers at C and D are having two scales which increases in opposite directions. It is used as seat for altitude bubble and also provides vernier reading for vertical angle measurement. The index arm can be rotated slightly with the help of clip screw for making permanent adjustments, if required. Glass magnifiers are fitted in front of each vernier to magnify the readings.

8. Telescope

The telescope is mounted on a horizontal spindle called horizontal axis or trunnion axis. The trunnion axis is supported at its two ends in bearings carried on standards. The telescope can be either rotated in horizontal plane or in vertical plane. The telescope may be of internal focussing or external focussing type. Internal focussing telescope is used in most of the transits.

9. Vertical Circle

Vertical circle is rigidly connected to the telescope and moves with it when the telescope is rotated about the horizontal axis. The vertical circle is graduated in degrees, with graduations at 20'. The graduations in each quadrant are numbered from 0° to 90° in opposite directions from the two zeros placed at the horizontal diameter of the circle. When the telescope is horizontal, the line joining the two zeros is horizontal.

Vertical angles on the vertical circle are measured by means of a pair of stationary verniers called C and D verniers, fixed diametrically opposite to each other. The least count of vernier is 20". With the help of a vertical circle clamp screw and its corresponding slow motion or tangent screw, the telescope can be set in any desired position accurately in vertical plane.

10. Altitude Bubble

A sensitive level tube placed on vernier frame is called altitude bubble. It is used to make horizontal axis truly horizontal. For measuring vertical angles transit is levelled with reference to altitude bubble.

11. Screws

A theodolite instrument has number of screws as its component parts. These are classified into different types depending on their functions.

a) Levelling Screws
b) Clamp Screws
c) Tangent Screws

a) Levelling Screws

These screws are used for levelling the instrument i.e., to make plate level axis truly horizontal.

The lower part of the foot screws are secured in the trivet by means of a ball-and-socket arrangement, and the upper threaded part passes through the threaded hole in the tribrach plate.

b) Clamp screws

These are used to fix the parts of a theodolite with which these are attached.

- Lower Plate Clamp Screw
- Upper Plate Clamp Screw
- Vertical plate Clamp Screw

Lower plate Clamp Screw : The clamp screw attached to the lower plate of a theodolite is called lower plate clamp screw. When it is tightened, the outer spindle gets fixed with the tribrach, and, thus, the lower plate gets fixed in position.

Upper plate Clamp Screw : The clamp screw attached with the upper plate of a theodolite is called upper plate clamp screw. When it is tightened, the inner spindle gets fixed with the outer spindle and, thus, the upper plate gets fixed in position.

The manipulation of the upper plate and lower plate clamp screws provide three conditions:

When both the upper plate clamp screw and the lower plate clamp screw are tightened, the instrument gets fully fixed.

When the upper plate clamp screw is tightened and the lower plate clamp screw is opened, the instrument rotates on its outer axis, there is no relative motion between the two plate and the readings in the horizontal vernier scales do not change.

When the lower plate clamp screw is tightened, and the upper plate is opened, the instrument rotates on the inner axis with outer axis fixed. The readings in the horizontal vernier scales change.

Vertical plate Clamp Screw : It is present on a frame fixed with standard and above the shaft of trunnion axis. It is used to clamp the telescope in any plane and hence at any desired vertical angle.

c) Tangent Screws

With each clamping screw, there is a tangent screw present in the instrument to provide fine movement. The tangent screws work only after its clamping screws get tightened. Thus when the upper clamp screw has been tightened, small movement of the upper plate can be made by the upper tangent screw, when the lower clamp screw has been tightened, small movement of the lower plate can be made by the lower tangent screw and similarly for vertical clamp screw.

12. Spindles

There are two spindles one inside the other. The inner spindle is conical solid and fits into the outer spindle which is hollow. The inner spindle is called upper axis since it carries the upper or vernier plate. The outer spindle is called lower axis, since it carries the lower or main scale plate. Both the axes have a common axis which forms the vertical axis of the instrument.

13. Tripod Stand

The theodolite is mounted on a strong tripod when being used in the field. The legs of the tripod are solid or framed. At the lower ends of the legs, pointed steel shoes are provided to get them pushed into ground. The tripod head has external screw to which the trivet of the levelling head is screwed.

Sometimes a circular box compass is mounted on the vernier scale between the standards. In modern theodolites, an adjustable trough compass or tubular compass can be fitted with a screw to the standard. The compass is provided for taking the magnetic bearing of a line.

1.5 TECHNICAL TERMS USED IN THEODOLITE SURVEYING

1. Centering

It is the process of setting up the theodolite exactly over the station mark. Centering may be done either by plumb bob or optical plummet.

2. Levelling

It is the process of making vertical axis of the instrument truly vertical.

3. Vertical axis

It is an axis about which the instrument can be rotated in a horizontal plane. It is also known as azimuth axis. It is the line passing through the common central axes of the inner and outer spindles.

4. Horizontal axis

It is an axis about which the telescope and the vertical circle can be rotated in a vertical plane. It is also known as trunnion axis, elevation axis or transverse axis.

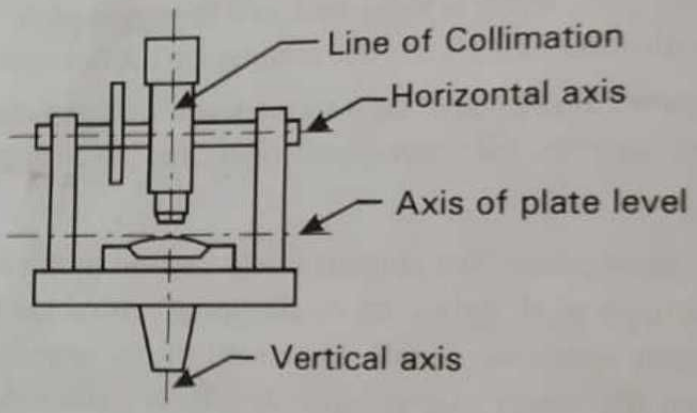


Fig. 1.1(a)

5. Axis of telescope

It is the imaginary line joining the optical centre of object glass and the centre of the eyepiece.

6. Line of sight or line of collimation

It is the imaginary line joining the intersection of the cross-hairs of the diaphragm and the optical centre of the object glass and its continuation.

7. Axis of plate level

It is the straight line tangential to the longitudinal curve of the plate level tube at its centre. It is horizontal when the bubble is centered.

8. Axis of the altitude level

It is the straight line tangential to the longitudinal curve of the altitude level tube at its centre. It is horizontal when the bubble is centered.

5. Transiting

It is the process of rotating the telescope over the horizontal axis through 180° in the vertical plane. It is also known as plunging or reversing.

6. Swinging the telescope

It is the process of turning the telescope about the vertical axis in a horizontal plane. The swing is called right swing when the telescope is turned clockwise and the swing is called left swing when the telescope is turned counter-clockwise.

7. Face left observation

It is an observation of horizontal or vertical angle when the vertical circle is to the left of the observer.

8. Face right observation

It is an observation of horizontal or vertical angle when the vertical circle is to the right of the observer.

9. Telescope normal

A telescope is said to be normal or direct when the vertical circle is to the left and the 'bubble up' if it is provided on the telescope.

10. Telescope inverted

A telescope is said to be inverted or reversed when the vertical circle is to the right and the 'bubble down'.

11. Changing face

It is an operation of bringing the face of the vertical circle, from (or) face left condition to face right condition, from face right condition to face left condition. This is achieved by transiting the telescope and then swinging it through 180°.

12. Double sighting or double centring

It is the operation of measuring an angle twice, once with telescope in the normal condition and another in the reverse condition.

13. A measure

It is the determination of the number of degrees, minutes and seconds or grades contained in an angle.

14. A Set of Observation

Finding horizontal angles once with face left observations and another with face right observations are called one set of observations.

15. Temporary adjustment

The setting of the theodolite over a station at the time of taking any observation is called temporary adjustment. This adjustment is necessary for every set up of the instrument.

1.8 USES OF THEODOLITE

Following are the various uses of a theodolite:

1. To measure horizontal angle and vertical angle.
2. To measure deflection angles and magnetic bearings.
3. For astronomical observations.
4. To run a straight line between two points.
5. For foundation layout of buildings, boundaries, etc.
6. To fix alignment of rail, road, electric poles, telephone poles, etc.
7. Extending a line in the field.
8. Taking observations for the measurement of areas and volumes.
9. Theodolite traversing.
10. Setting out roads and railway curves, grade lines.
11. Finding the difference of elevations of inaccessible points.
12. Finding the vertical height of an object, etc.

1.9 FUNDAMENTAL LINES OF A THEDOLITE

The fundamental lines are imagined in a theodolite instrument (fig.1.2) are

1. The Vertical Axis.
2. The Horizontal axis or trunnion axis or transit axis.
3. The Line of collimation or Line of sight.
4. Axis of the altitude level tube.
5. Axis of the plate level.

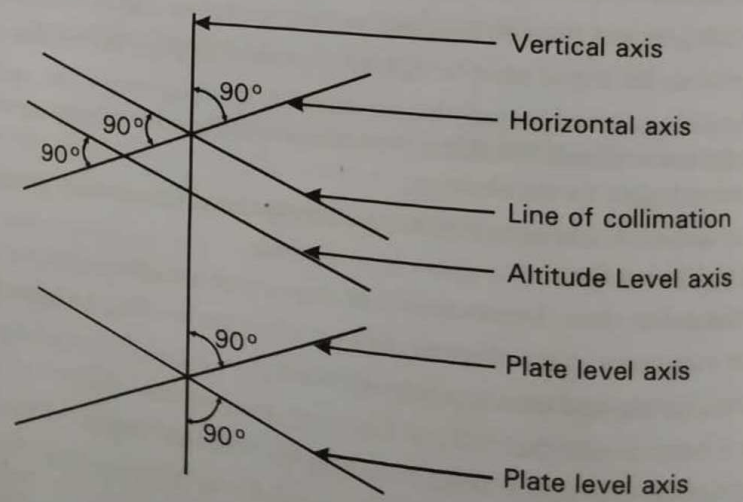


Fig. 1.2 : Fundamental Lines of a Th

Relations among Fundamental Lines

A theodolite in proper adjustment must have the following conditions or relations between fundamental lines.

1. Axis of plate level must be perpendicular to vertical axis.
2. Horizontal axis must be perpendicular to vertical axis.
3. Line of collimation must be perpendicular to the horizontal axis.
4. Axis of altitude level must be parallel to the line of collimation.
5. Vertical circle vernier must read zero when the line of collimation is horizontal.

1.10 ADJUSTMENTS OF A THEODOLITE

A theodolite should be properly adjusted to obtain accurate observations. The adjustments are mainly of two types. They are

- Permanent adjustments and
- Temporary adjustments.

Adjustments made to establish fixed relations between the fundamental lines of a theodolite are known as permanent adjustments. These adjustments once made remain permanent for a very long period.

Adjustments made at each setting of the instrument before taking observations are known as temporary adjustments.

1.11 TEMPORARY ADJUSTMENTS OF A THEODOLITE

These are the adjustments which are done before making the observations at every setup of the instrument. They consist of setting up the theodolite, levelling it, and focussing the eyepiece and objective.

These temporary adjustments or station adjustments are made to achieve the following *two* conditions:

1. The plane of collimation is horizontal.
2. The vertical axis passes through the centre of the peg.

Before setting up the theodolite it should be ensured that:

a) The levelling screws are at the centre of their run.

b) The shifting head of the theodolite is at its centre so that equal movement is possible in all the directions.

c) The wing nuts on the tripod legs are tight enough so that when raised, the tripod legs do not fall under their own weight.

Temporary adjustment of a vernier theodolite consists of following operations:

1. Setting
2. Centring
3. Levelling and
4. Focussing.

1. Setting

The setting operation consists of fixing the theodolite with the tripod stand along with approximate levelling and centring over the station.

1. For setting up the instrument, the tripod is placed over the station with its legs widely spread so that the centre of the tripod head lies above the station point and its head approximately level (by eye estimation).
2. The instrument is then fixed with the tripod by screwing through trivet.
3. The height of the instrument should be such that observer can see through telescope conveniently.
4. After this, a plumb bob is suspended from the bottom of the instrument and it should be such that plumb bob should point near to the station mark.

2. Centring

1. The operation involved in placing the vertical axis of the instrument exactly over the station mark is known as centring.
2. For centering the instrument over a station, lift the instrument bodily without disturbing the relative positions of legs and move it until the plumb bob is approximately over the station peg. Then move the legs radially and circumferentially. (Radial movement helps in centering and circumferential movement helps in approximate levelling). Thus bring the plumb bob with in a circle of diameter 10mm over the station point.
3. Unscrew the clamping ring of the shifting head, slide the instrument until the plumb bob is exactly over the station mark and tighten the clamping ring of the shifting head.

3. Levelling

Levelling of an instrument is done to make the vertical axis of the instrument truly vertical. Generally, there are three levelling screws are present in a theodolite instrument. Thus, levelling is being achieved by carrying out the following steps (Fig. 1.3):

1. Loosen the upper clamp screw. Turn the upper plate until the longitudinal axis of the plate level is approximately parallel to the line joining of any two foot screws.
2. Bring the bubble of the plate level to the centre of its run by turning the foot screws (1) and (2) simultaneously either towards or away from each other.
3. Turn the upper plate through 90° so that it is at right angles to the original position.
4. The plate level is now parallel to the line joining the foot screw (3) and the centre of the line joining the foot screws (1) and (2).
5. Bring the bubble to the centre by turning the screw (3) alone.

6. Turn the instrument back to its original position by rotating back through 90°. Bring the bubble to the centre by turning screws (1) and (2).
7. Repeat steps (3) and (4) till the bubble remains central in both positions.
8. Turn the instrument through 180° so that it is again parallel to the line joining the foot screws (1) and (2). If the instrument is in adjustment the bubble will remain central. If not, it needs permanent adjustments.

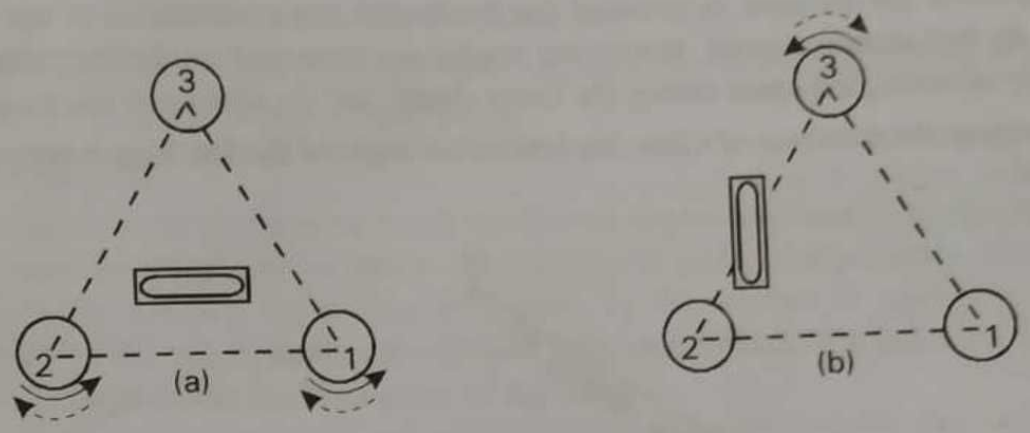


Fig. 1.3 : Levelling

4. Focussing

It consists of focussing the eyepiece and the objective.

Focussing the eyepiece: This operation is done to make the cross-hairs appear clearly visible. The following steps are involved:

1. The telescope is directed towards the sky or a sheet of white paper held in front of the objective.
2. The eyepiece is moved in or out until the cross-hairs appear clear and distinct.

Focusing of eye-piece depends on the eye-sight of observer and so for each observer it needs to adjusted accordingly.

Focussing the objective: It is done for each independent observation to bring the image of the object in the plane of cross hairs.

The following steps are involved:

1. The telescope is directed towards the object.
2. The focussing screw is turned until the image appears clear and sharp.

If focusing has been done properly, there will be no parallax i.e., there will be no apparent movement of the image relative to the csross hairs, if the observer moves his eye from one side to the other or from top to bottom.

1.12 MEASUREMENT OF DIRECTION USING THEODOLITE

The different ways by which the direction of a line is depicted in surveying are computed either from horizontal angles or from vertical angles or both. Thus, primary elements of observation during surveying are the horizontal angles and the vertical angles. These quantities can be observed directly in the field using theodolite.

1.13 METHODS OF MEASURING HORIZONTAL ANGLES

a) Direct method,

b) Repetition and

c) Reiteration.

a) Direct method of measuring horizontal angle (fig.1.4)

A theodolite can be used to measure the horizontal angle subtended at the center of the instrument by two station signals. Horizontal angles are measured on the horizontal circle of a theodolite by operating the upper clamp, the lower clamp, and the upper and lower tangent screws.

To represent the direction of a line, the horizontal angle of the line from a reference line is to be measured.

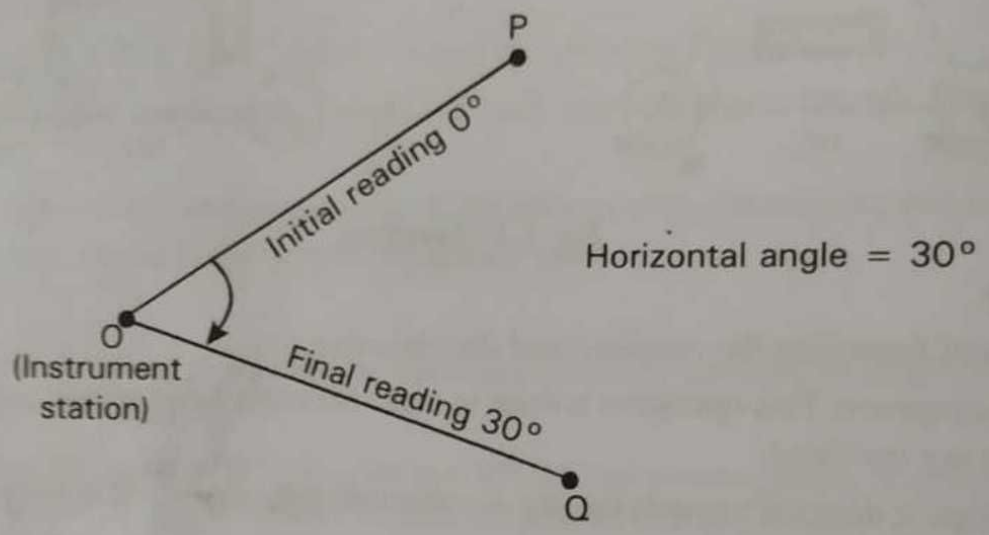


Fig. 1.4 : Horizontal angle

The steps required to be adopted are as follows:

1. Two points one on each of the lines, say P and Q, are to be marked.
2. A transit theodolite is to be set at the point of intersection of the lines, say at O. Initially, the instrument is in the face left condition and its temporary adjustment is to be done over the point O.
3. Both the lower and upper plate main screws are to released and get the vernier A set to 0° (or 360°) mark on the main scale. After clamping the upper main screw, index of vernier A is to be brought exactly to the zero of the main scale using the upper plate tangent screw.
4. At this stage the reading of the vernier B should be 180°.
5. Swing the telescope in the horizontal plane and point it to the left station, say P. Tighten the lower plate clamp screw, and bisect the signal at P exactly using the lower plate tangent screw.
6. Loosen the upper plate main screw and turn the telescope the signal at Q is sighted. Tighten the upper clamp screw and bisect the ranging pole at Q exactly using the upper plate tangent screw.

7. Read both the verniers A and B and record the readings. The reading of the vernier A is the angle POQ. The vernier B gives the value of angle POQ after deducting from it 180°. The mean of two values of the angles obtained from the verniers A and B is the required angle POQ.
8. Change the face of the instrument to the face right by transiting the telescope and swinging it by 180°.
9. Repeat steps 3 to 8 and determine another value of the angle POQ.
10. The mean of the face left and face right observations is the final required angle POQ. Fig. 1.4 shows typical horizontal angle measurement.

b) Repetition method

This method is used to measure small horizontal angles accurately. In this method, the horizontal angle is measured several times and the value is added mechanically. The horizontal angle is obtained by dividing the accumulated value by the number of repetitions. Usually 3 repetitions with face left and 3 repetitions with face right are adopted. The precision thus attained is to a much finer degree than the least count of the vernier.

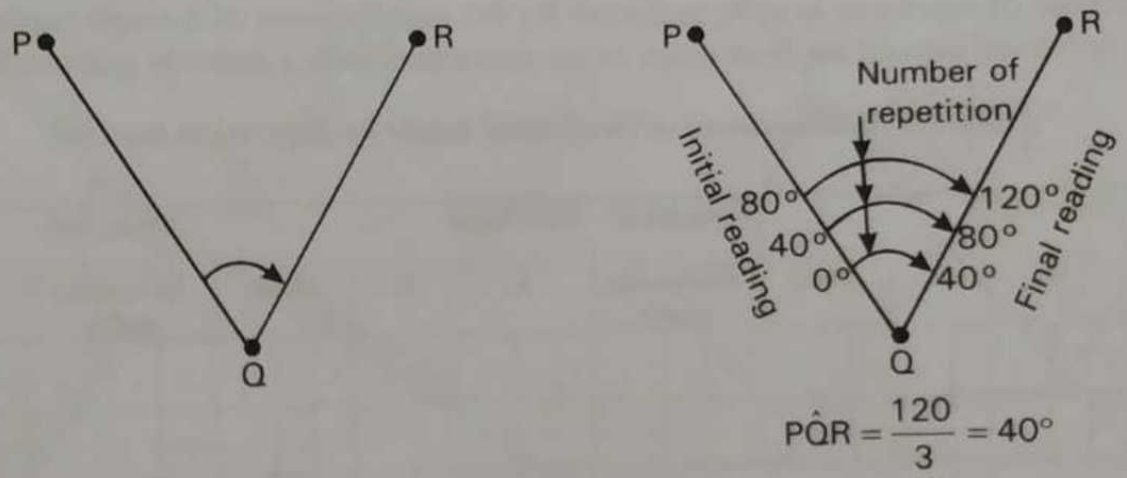


Fig. (a) *Fig. (b)*

Fig. 1.5 : Repetition method

The steps involved in the measurement of the horizontal angle, say PQR (fig.1.5 (a)) by method of repetition are as follows

1. Set up the instrument over the station Q and do all temporary adjustments. Keep the vertical circle to the left.
2. Set the vernier A to zero with the help of upper clamp and tangent screws. Note the readings of vernier A and B.
3. Loosen the lower clamp and turn the telescope towards the left signal at P. Clamp the lower clamp and bisect the point P exactly using lower tangent screw.
4. Loosen the upper clamp and turn the instrument clockwise to bisect the signal at R. Clamp the upper clamp and bisect R exactly using upper tangent screw.
5. Read both verniers to get approximate value of the angle PQR.

6. Unclamp the lower plate and turn the telescope clockwise to sight the signal P again. Clamp the lower clamp and bisect P exactly using lower tangent screw. Check that the vernier A and B readings have not changed.
7. Loosen the upper plate and turn the telescope clockwise and again bisect R. Clamp the upper plate bisect R exactly using upper tangent screw. The vernier will now read twice the value of angle PQR.
8. Repeat the steps (6) and (7) once again. The final reading of the vernier A will be thrice the angle PQR.
9. Read both verniers. Divide the final reading by three to get the value of angle PQR.
10. Change the face of the instrument to right face. Repeat steps (2) to (9). Determine another value of the angle PQR.
11. Determine average value of the two horizontal angles obtained with face left and face right. This is the precise value of angle PQR.
12. Record all the observations in the tabular form (Table-1.1).Fig.1.5(b) shows the typical example for angle measured by this method.

Note : Method of repetition is to be preferred for the measurement of a single angle and when accuracy is desired beyond the least count of the instrument with a coarsely graduated circle.

Table-1.1 Measurement of horizontal angle by Repetition method

Inst. at	Sighted to	Face left									Swing Right			Face Right									Swing Left			Average Horizontal angle		
		A			B		Mean			Horizontal angles			A			B		Mean			Horizontal angles					
		°	'	"	'	"	°	'	"	°	'	"	°	'	"	'	"	°	'	"	°	'	"	°	'	"

Advantages of Repetition method

The following errors are eliminated by repetition method:

1. Taking reading on both the verniers eliminates the errors due to eccentricity of verniers and centres.
2. Errors due to imperfect adjustment of instrument are eliminated by taking face left and face right observations.
3. Taking reading at different parts of the circle, eliminates the errors due to inaccurate graduations.
4. Errors due to inaccurate bisection of the signal are eliminated as they tend to balance each other.
5. Other errors are also minimized as the sum is divided by the number of repetitions.

The errors due to the following are **not eliminated** as all these errors are of cumulative nature:

1. Slip,
2. Displacement of station signal,
3. Verticality of the vertical axis.

c) Reiteration method

This method is suitable when several angles are measured from a single station. Several angles are measured successively from the initial station. The angle between last and first station is also measured and thus horizon is closed.

The process of measuring angles at an instrument station around the point to obtain a check on their sum which should be equal to 360° is called closing the horizon. When the horizon is closed, the final reading should be the same as initial reading. If there is any error within permissible limit, it can be distributed equally among all the angles. If the error is large, the whole field work will be repeated.

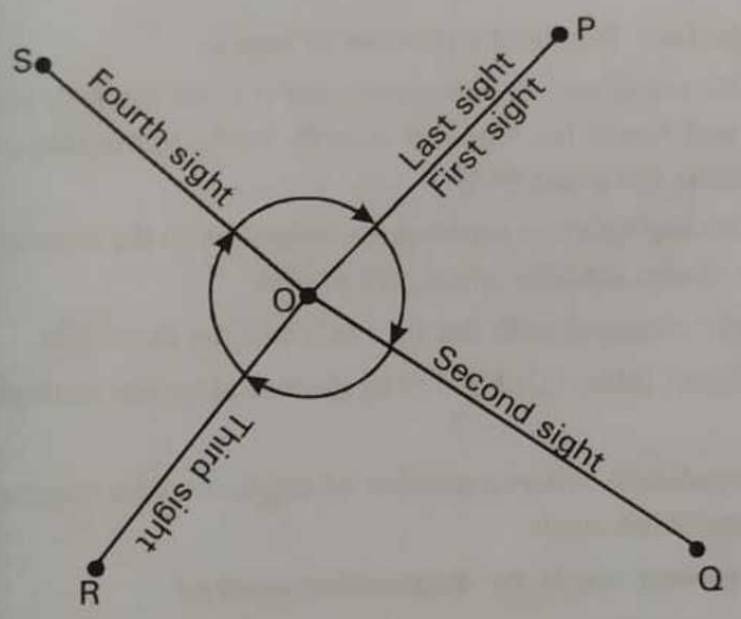


Fig. (a)

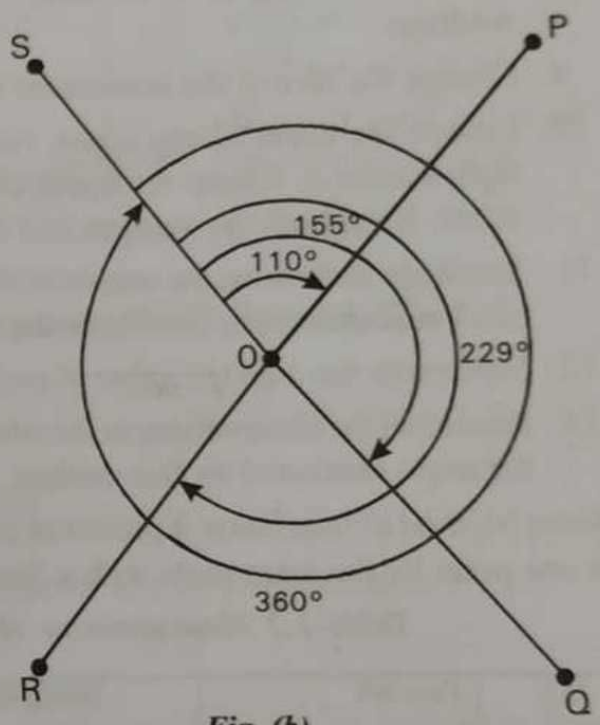


Fig. (b)

SOP

$$B\hat{O}A = 110°$$

$$P\hat{O}Q = 155° - 110° = 45°$$

$$Q\hat{O}R = 229° - 155° = 74°$$

$$R\hat{O}S = 360° - 229° = 131°$$

Fig. 1.6 :. Reiteration method

Fig.1.6(a) shows the instrument station O where the angles POQ, QOR, ROS and SOP have to be measured by reiteration. The following procedure is used.

1. Set up the instrument over the station O and do all temporary adjustments. Keep the vertical circle to the left.

2. Set the vernier A to zero using upper clamp and its tangent screw.
3. Loosen the lower clamp and direct the telescope to signal at P. Clamp the lower clamp and bisect P accurately using lower tangent screw. Read both verniers.
4. Loosen the upper clamp and turn the telescope clockwise until the signal at Q is bisected. Clamp the upper clamp and bisect Q exactly using upper tangent screw.
5. Read both verniers. Mean of the vernier readings gives the horizontal angle POQ.
6. Loosen the upper clamp again and turn the telescope clockwise until the signal at R is bisected. Use the upper tangent screw for exact bisection. Read both verniers and determine the angle QOR. The angle QOR is obtained by finding the difference between the readings to R and Q.
7. Similarly, determine the angle ROS.
8. Finally, sight the signal P and read both the verniers. Vernier A should now read 0° or (360°). If not, note the reading and find the error due to slip, etc. If the error is small, distribute it equally to all angles. If the error is large, repeat the above procedure and take fresh set of readings.
9. Change the face of the instrument to right face. Repeat the process in step 2.
10. Loosen the upper clamp screw, rotate the telescope counterclockwise (i.e. swing left) and sight station S. Clamp the upper clamp and bisect the signal S exactly using upper tangent screw. Read both the verniers and determine the angle POS.
11. Similarly, determine the angles SOR, ROQ and QOP by rotating the telescope in the counter-clockwise direction. Distribute the error, if any, equally among all angles.
12. Determine the average value of each angle obtained with the face left and the face right.
13. Record all the observations in the tabular form (Table-1.2).Fig.1.6(b) shows the typical example for angle measured by this method.

Note: Method of reiteration is preferred in triangulation, where a number of angles may be required at one point by the instrument with a finely graduated circle.

Table-1.2 Measurement of horizontal angle by Reiteration method

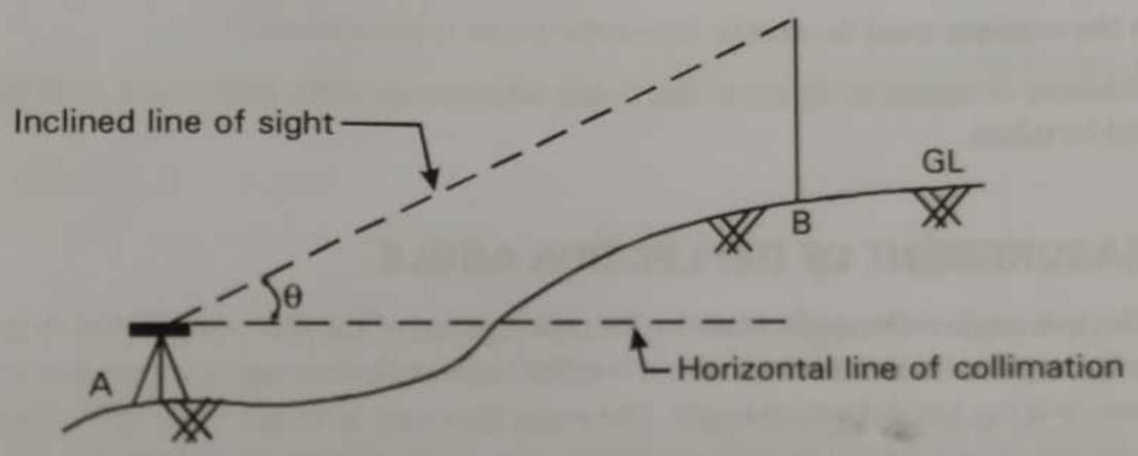
Inst. at	Sighted to	Face left									Swing Right			Face Right									Swing Left			Average Horizontal angle		
		A			B		Mean			Horizontal angles				A			B		Mean			Horizontal angles					
		°	'	"	'	"	°	'	"	°	'	"		°	'	"	'	"	°	'	"	°	'	"		°	'	"

1.14 MEASUREMENT OF VERTICAL ANGLES

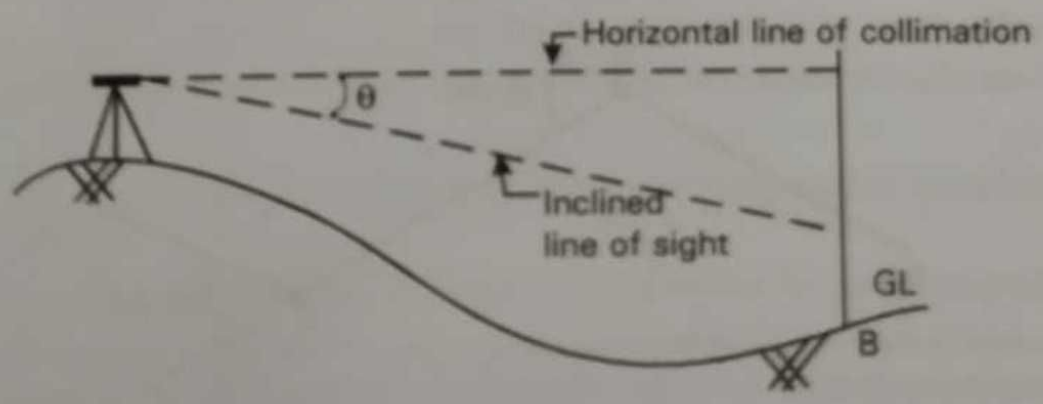
A vertical angle is defined as the angle subtended by the inclined line of sight and the horizontal line of sight in the vertical plane. If the point sighted is above the horizontal axis of the theodolite, the vertical angle is known as an angle of elevation, and if it is below it is known as an angle of depression.

To measure a vertical angle subtended by the station B at the instrument station A (fig.1.7), the following steps are involved:

1. The temporary adjustment of the instrument is to be done on the station A.
2. Then, levelling of theodolite is to be done using altitude level (the operations involved are same as levelling using plate level).
3. Loosen the vertical circle clamp, and direct the telescope towards the object whose vertical angle is required to be measured. Clamp the vertical circle, and bisect the point by turning the vertical tangent screw.
4. Read and record the scale with vernier C and D in Table-1.3.
5. Change the face of the instrument and read the vertical angle again.
6. The required vertical angle is the average of the values in steps 4 and 5.



(a) Angle of elevation



(b) Angle of depression

Fig. 1.7 : Measurement of Vertical angle

Table-1.3 Measurement of Vertical angle

Inst. at	Sighted to	Face left									Face Right									Vertical angles			Average Vertical angle		
		C			D			Mean			C			D			Mean								
		°	'	"	°	'	"	°	'	"	°	'	"	°	'	"	°	'	"	°	'	"	°	'	"

While measuring vertical angles, the following precautions must be taken:

1. For computing the reduced levels of the objects, it is essential to know whether the measured vertical angle is the angle of elevation or depression. Therefore, care should be taken to note +ve sign for angles of elevation and -ve sign for angles of depression.
2. Both the verniers must be read to obtain the mean vertical angle.
3. To eliminate or minimise the errors due to non adjustments of the instrument, both face readings should be taken.

1.15 MEASUREMENT OF DEFLECTION ANGLE

A deflection angle is the angle, made by the prolongation of the preceding line with the following line. When the angle is measured clockwise it is called right deflection angle whereas when measured anticlockwise it is the left deflection angle. The angle may vary between 0° to 180°. This method of measuring angles is very useful in open traversing, e.g., in the alignment of highways and railways.

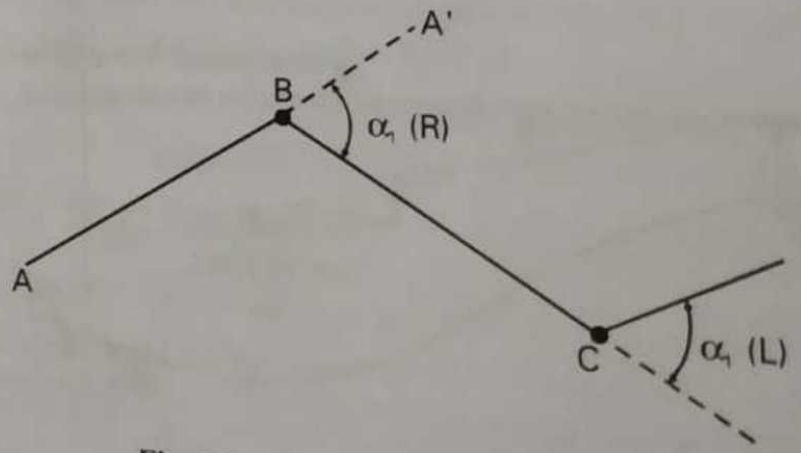


Fig. 1.8 : Measuring deflection angles

1. Suppose it is required to measure the deflection angle B (Fig.1.8). Set up and level the instrument over station B.
2. Set the vernier to read zero.
3. Take a back sight on station A and clamp both the plates.

4. Transit the telescope so that the line of sight is in direction of AB produced (BA'). Ensure that the verniers read zero.
5. Unclamp the upper plate, swing the telescope clockwise to sight station C. Read both the verniers. The mean of the two readings gives the approximate value of the deflection angle at B.
6. Loosen the lower clamp and again sight back on A. The readings on the verniers will be unchanged.
7. Transit the telescope. Unclamp the upper plate and sight C. Read both the verniers and find the mean value.
8. The deflection angle thus measured is doubled. One half value of this average value is the value of the deflection angle at B.

1.16 ERRORS IN THEODOLITE WORK

Errors which occur in theodolite surveying are classified into three types depending upon their sources.

1. Instrumental errors
2. Personal errors
3. Natural errors
4. Mistakes

1. Instrumental Errors

Errors due to imperfections and/or non-adjustment of instrument are all systematic, and these can either be eliminated or reduced to a negligible amount by adopting appropriate methods. The instrumental errors involved in theodolite surveying are

a) Errors due to imperfect adjustments of plate level

Due to this error, vertical axis of the instrument will not be truly vertical even if the plate bubble is brought to the centre. Consequently, the horizontal and vertical angles measured will not be correct.

To eliminate this error, the instrument can be levelled with reference to altitude level, provided if it is in adjustment.

b) Error due to line of collimation not being perpendicular to the horizontal axis

Due to this error, the telescope will not revolve in a vertical plane when it is raised or lowered. Instead, it will trace out the surface of a cone. This will cause error in the measurement of horizontal angle between the points at different elevations.

This error can be eliminated by taking face left and face right observations.

c) Error due to horizontal axis not being perpendicular to vertical axis

Due to this error, the line of sight moves in an inclined plane when the telescope is raised or lowered. Consequently, the horizontal and vertical angles measure will not be correct.

This error can be eliminated by taking face left and face right observations.

d) Error due to imperfect adjustment of the vertical circle vernier

This error occurs if the C and D verniers do not read zero when the line of sight is horizontal. Due to this the measured vertical angles will not be correct. This error is called Index error.

This error can be eliminated by taking mean of the vertical angle on both faces or by applying index corrections.

e) Error due to eccentricity of inner and outer axes

This error occurs, when the centres of horizontal scale plate and vernier plate do not coincide with the common centre.

This error can be eliminated by taking mean of the A and B vernier readings.

f) Error due to eccentricity of verniers

This error occurs if the zeros of A and B verniers are not at the ends of the same diameter and the difference between A and B vernier is not equal to 180°.

This error can be eliminated by taking both vernier readings and taking their mean value.

g) Error due to imperfect graduations on the horizontal scale

This error occurs if the graduations on the horizontal scale are not uniformly spaced.

This error can be eliminated by taking observations over different parts of the horizontal scale.

2. Personal errors

Personal errors can be classified into two types.

- Manipulation errors
- Sighting and reading errors

Manipulation Errors

a) Errors due to inaccurate centering

If the instrument is not centred properly, there will be error in the horizontal angles. This can be eliminated by centering the instrument exactly over the station point.

b) Error due to inaccurate levelling

This error occurs when the level bubble is not centred correctly. This error can be eliminated by levelling carefully.

c) Slip in screws

This error occurs when the clamp screws are not properly tightened. This error is of serious one as the direction of line of sight changes. This error can be eliminated by properly tightening clamp screws and taking right swing as well as left swing observations.

d) Improper use of tangent screws

This error is introduced when the tangent screws are improperly used.

This is a common error and can be eliminated by using correct tangent screw carefully.

Sighting and reading errors

a) Inaccurate vernier reading

This error occurs when the vernier readings are not taken accurately. This error is eliminated by using the method of repetition.

b) Inaccurate sighting

This error occurs if the station mark is not bisected accurately by vertical cross hair.

This error can be eliminated by sighting the lowest point of the ranging rod or arrow at the station point.

c) Parallax

This error occurs if eyepiece and objective are not focussed properly. To avoid this error, eyepiece and objective are properly focussed while sighting.

3. Natural Errors

Errors due to natural causes are as given below:

a) Temperature Effect

Temperature effect causes unequal expansion of various components of theodolite. The unequal exposure of different parts to sun may be avoided by using surveyor's umbrella.

b) Wind Effect

Vibration of different parts due to wind will also cause errors. Best way to minimize such errors will be to suspend the work when the wind is strong.

c) Refraction Effect

Bright sunlight & refraction effects causes accurate sighting difficult. To avoid refraction effect of line of sights, the sights very close to ground, close to bushes, etc., should be avoided.

d) Unequal Settlement of Tripod

If the ground is soft, unequal settlement of tripod causes error. To avoid it select firm ground for setting the instrument, press the tripod legs sufficiently before completing temporary adjustment or use triangular frame.

4. Mistakes

Mistakes occur during reading or laying direction of line due to carelessness of the observer. Some of them are:

- Forgetting to level the instrument.
- Reading the wrong vernier scale.
- Misreading the vernier.
- Dropping one division of the main scale reading say 20'.
- Reading the vernier in the wrong direction.
- Turning the wrong tangent screws.
- Failing to tighten the clamp screw.
- Reading the numbers on the horizontal scale from the wrong row.

- Reading angles in the wrong direction.
- Sighting on the wrong signals or setting up over the wrong station.
- Booking the wrong values.
- Missing the wrong right or left in deflection angle.
- Missing sign (+ or -) for vertical angles.
- Using the stadia instead of the cross-hair.
- Using haphazard field procedure.

1.17 ELIMINATION OF ERRORS

Eliminating each error is already discussed in above topics, for quick remembrance following summary is given:

Instrumental Errors

Errors due to instrumental imperfections and/or non adjustment are all systematic and can be eliminated by obtaining the mean of two values one observed before and the other after a reversal of the horizontal plate by plunging the telescope and rotating it about the vertical axis. Further, the error in either horizontal or vertical angle due to inclination of the vertical axis can be eliminated, by levelling the plate bubbles again in addition to the reversal of the plate.

Personal errors

The personal errors are random and hence cannot be eliminated. They form a large part of the resultant error in transit and theodolite work. Of the personal errors, those due to inaccuracies in reading and setting the vernier or reading and setting of the optical micrometer and to not sighting exactly on the point are likely to be of greater magnitude.

Natural Errors

Natural errors are generally random, but under certain conditions systematic errors may arise from natural causes. On surveys of very high precision, special attempt is made to establish a procedure which will as nearly as possible eliminate natural systematic errors.

THEODOLITE TRAVERSING

1.18 INTRODUCITON

Traversing is one of the simplest and most popular methods of establishing control networks in engineering survey.

A traverse consists of a series of straight lines connected successively at established points, along the route of a survey. The points defining the ends of the traverse line are called traverse stations or traverse points. Distances between traverse stations are known as traverse side and are measured either by direct measurement using a Tape or Electronic Distance Measuring (EDM) equipment, or by indirect measurement using tacheometer. At stations where a traverse side changes its direction, relative direction are measured with a or theodolite.

Traverse networks have the following advantages:

1. Little reconnaissance is required compared with that needed for an interconnected network of points.
2. Observations only involve three stations at a time so planning the task is simple.
3. Traversing may permit the control to follow the route of a highway, pipeline or tunnel, etc., with the minimum number of stations.

1.19 TYPES OF TRAVERSE

1. Open Traverse
2. Closed Traverse.

1. Open Traverse

An open traverse originates from a point whose position may be known or unknown but terminates to a point whose position is not known (fig. 1.9).

- The open traverse is considered to be the least desirable type of traverse, because it provides no check on the accuracy of the starting control or the accuracy of the fieldwork. For this reason, traverse is never deliberately left open.
- Open traverse is used only open projects like roads, cannels, railway lines, shoreline protection, canal, boundary of river, pipe lines, etc.

- To reduce error each distance should be measured in both directions and also should roughly checked by using the stadia hairs of the theodolite and angles should be measured by method of repetition and should also checked by magnetic bearings.
- When an open traverse is used the work should be checked by providing cut off lines and by making observations on some prominent points visible form as many stations as possible.

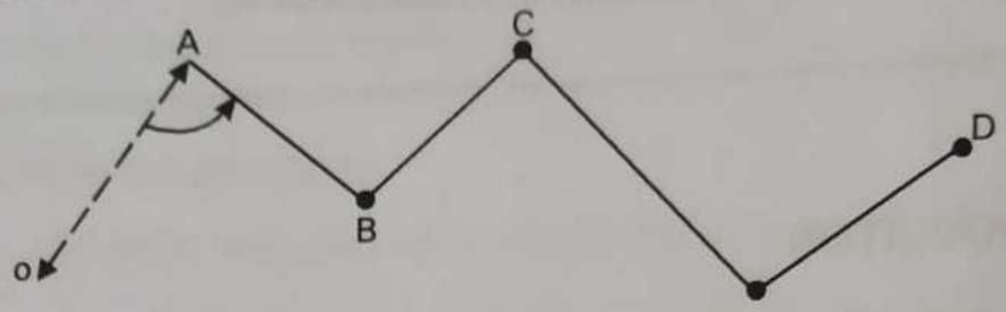


Fig. 1.9 : Open traverse

2. Closed Traverse

When a traverse originates from a known position and also terminates to known position then it is called a closed traverse.

There are two types of closed traverse

- Closed on the starting point and
- Closed on a second known point.

Closed loop traverse

This type of closed traverse begins at a point of known control, moves through the various required unknown points, and returns to the same point (fig.1.10(a)).

- This type of closed traverse is considered to be the second best and is used when both time for survey and limited survey control are considerations.
- This type of traverse permits an internal check on the accuracy of angular measurements, provides an indication of the consistency of measuring distances as well as angles. But detection of systematic errors in linear measurement or errors in the orientation of the traverse is not possible.
- This type of closed traverse is known as "geometrically and mathematically closed".
- This type of traverse is recommended for minor projects.

- Linear measurements and
- Angular measurements.

Field Measurements

Linear measurements

The length of traverse sides are measured by direct method using tape or EDM or by indirect method using tacheometer. However, the choice of the method depends on the method of traversing employed. The checking of the linear measurements consists in taping each traverse line a second time, preferably in the reverse direction on different dates and by different parties.

Angular measurements

The direction of traverse sides are measured by deflection angles, interior angles, azimuth etc. Immediately after completion of a set of observation, an arithmetical check on the angular error of closure is required to be performed in the field itself to detect any blunder or excessively large error in angular measurement.

Computations

The traverse computations involve

- Calculation of consecutive coordinates of traverse stations,
- Checking in error of closure,
- Determination of the amount of closing error,
- Adjustment of traverse by balancing of consecutive coordinates,
- Calculation of independent coordinates and
- Determination of corrected distances and azimuth of sides.

1.22 LINEAR AND ANGULAR MEASUREMENTS

After fixing the traverse stations, the measurement of length of the traverse lines and angles or directions proceeds simultaneously. The choice of instrument for making measurements depends upon the traversing method employed.

Linear measurements

The distance between two adjacent traverse stations can be measured by one of the following methods:

- Taping or chaining.
- Stadia or Tacheometric method.
- EDM method.

Angular measurements

The following are the two main methods of traversing adopted to measure the directions of survey lines.

1. Direct measurement of angles between successive lines
2. Fast needle method

The first method is generally used for long traverse or where high precision is required while second method is used for short traverse where great precision is not required and for topographical surveys.

Traversing by direct measurement of angles

This method is usually adopted for long traverses where high degree of accuracy is prime consideration. In this method the angles between successive lines and the magnetic bearing of initial line are measured in the field. The bearings of all other lines are determined from the bearing of initial line and the measured angles. The lengths of traverse lines are measured either by a chain or tape.

The angles between the successive lines are measured by any one of the following methods.

a) By included angles,

b) By deflection angles,

c) By direct angles.

Traversing by included angles

This method is suitable for land surveying. The method is preferred for a closed traverse. In this method, included angle between two successive lines are measured. The included angles measured may be interior or exterior angle accordingly as the traverse is run in counterclockwise or clockwise direction. Generally the traverse is run in a counter-clockwise direction. When great accuracy is required angles may be measured by repetition method.

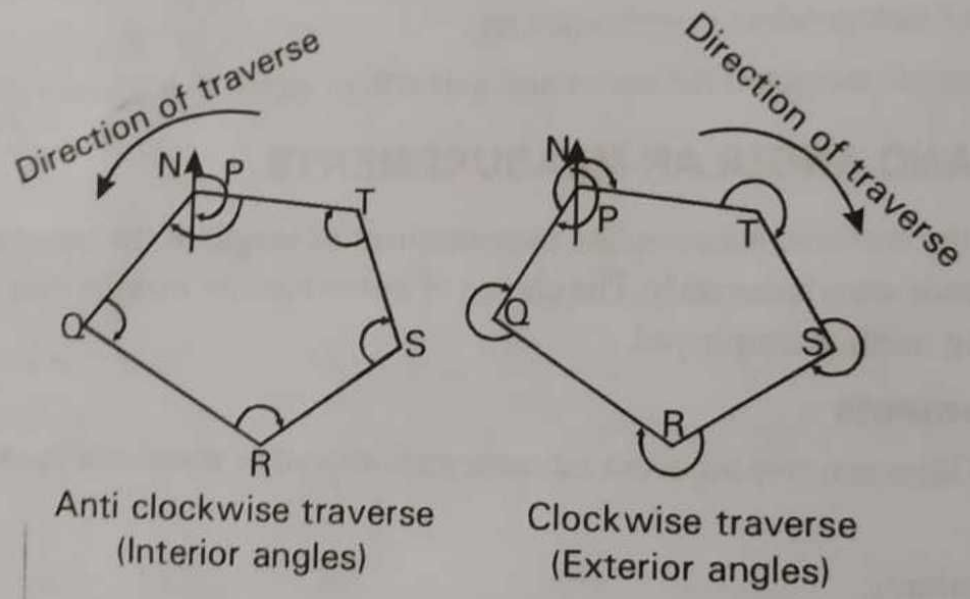


Fig. 1.12

The following procedure is used (Refer fig. 1.12).

1. Set up the instrument over the starting station P and do all temporary adjustments.
2. Orient the telescope in the magnetic meridian direction and then determine the magnetic bearing of the initial line PQ.
3. Set the vernier A to zero using upper clamp and tangent screw. (Keep face left). Take a back sight on last station T using lower clamp and tangent screw. Vernier A still reads zero.

4. Loosen the upper clamp. Turn the telescope clockwise and take a foresight on Q. Clamp the upper plate and bisect Q exactly using upper tangent screw. Read both verniers. The mean of the two verniers is the required included angle TPQ.
5. Change the face and again measure angle TPQ.
6. The average of the two included angles in steps 4 and 5 is the better included angle TPQ. Note: For greater accuracy, measure the angle by repetition method.
7. Shift the instrument to second station Q. Measure the included angle PQR by taking a back sight on P and a foresight on R as done in steps (3) and (4).
8. Similarly, measure included angles at R, S and T.
9. Measure the lengths of traverse lines PQ, QR, RS, ST and TP using a tape or chain.
10. Locate the details such as inaccessible objects, roads, buildings, etc., using theodolite or tape only or using both theodolite and tape.

Traversing by deflection angles

Deflection angle is the angle which a survey line makes with the prolongation of the previous line. It is designated Right (R) or Left (L) accordingly as it is measured clockwise or counter-clockwise from the prolongation of previous line. Its value varies from 0° to 180°.

This method is used for open traverse conducted for the survey of roads, railways, canals, etc., where the traverse lines make small deflection angles.

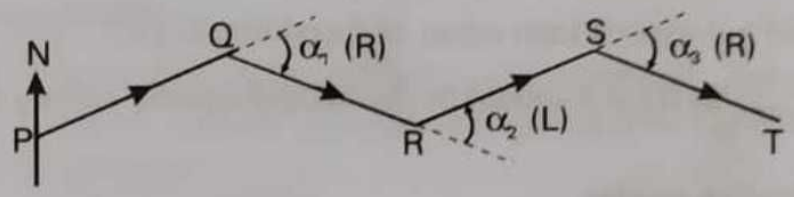


Fig. 1.13 : Traverse with deflection angles

The following procedure is used (Refer fig. 1.13).

1. Set up the instrument over the starting station P and do all temporary adjustments.
2. Measure the magnetic bearing of the line PQ.
3. Shift the instrument to station Q. Set vernier A to zero using upper clamp and its tangent screw.
4. Take a back sight on P using lower clamp and tangent screws. The vernier A still reads zero.
5. Transit the telescope. Now the line of sight is in the direction of PQ produced.
6. Loosen the upper clamp and turn the telescope clockwise to take a foresight on R. Read both verniers. Find the mean of the two verniers (α_1). Also note the direction of deflection angles which is to the right, i.e., $\alpha_1 \rightarrow R$.
7. Change the face and again measure the deflection angle at Q.
8. Average of two values in steps 6 and 7 is the required deflection angle at Q. (i.e. $\alpha_1 \rightarrow R$).

9. Repeat the above procedure at all stations.
10. Measure the lengths of traverse lines and locate the details.

1.23 FIELD CHECKS IN THEODOLITE TRAVERSE

Checks in a closed traverse

A closed traverse should be checked for errors in linear and angular measurements. The following are the checks for linear and angular measurements done in the field.

1. Checks for linear measurements

To avoid errors in linear measurements, the measurements should be carried out by different parties on different dates in both the directions. Then checks for latitudes and departures should be applied, i.e., $\sum Latitude = 0$ and $\sum Departure = 0$ Any closing error indicates error in linear measurements which should be balanced.

2. Checks for angular measurements

a) Traversing by included angles

- Sum of measured interior angles should be equal to (2n - 4) right angles, where, n is Number of sides of the traverse.
- Sum of measured exterior angles should be equal to (2n + 4) right angles.
- Error if any should not exceed least count of theodolite $\times\sqrt{n}$.

If the error is within this limit it should be distributed equally among all the angles to satisfy the above conditions.

b) Traversing by deflection angles

Sum of deflection angles should be equal to 360°, considering right deflection angles as positive and left deflection angles as negative.

c) Traversing by direct observation of bearings

The difference between fore bearing and back bearing of the same line should be 180°.

Checks in an open traverse

Angular and linear measurements in an open traverse cannot be checked as a whole directly. The following indirect methods are commonly used to check an open traverse in parts.

1. Running cut off lines

In this method, cut off lines are run between certain intermediate stations. Fig.1.14 shows an open traverse PQRSTUV having cut off lines PS and SV. Considering PQRS and STUV as small closed traverses, fore and back bearings of lines PS and SV may be checked. By measuring the length of cut off lines PS and SV, apply checks for closed traverse, i.e., $\sum Latitude = 0$ and $\sum Departure = 0$.

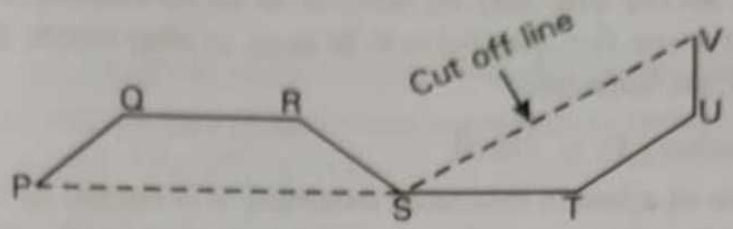


Fig. 1.14 : Running cut off lines

2. Sighting a prominent object

In this method, if there is any prominent object on either side of the traverse, it can be sighted for checking the open traverse. Fig.1.15 shows an open traverse PQRSTUV with two prominent points M and N. The bearings and lengths of PM, QM, RM, SM and SN, TN, UN, VN may be observed in the field to check the open traverse.

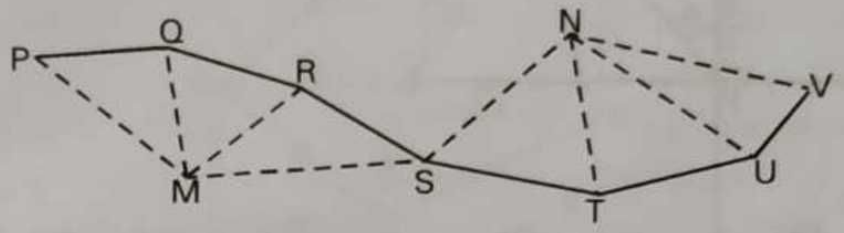


Fig. 1.15 : Sighting a prominet object

3. Astronomical observations

Open traverse can also be checked by astronomical observations. Here, true bearing of certain lines determined by astronomical observation are compared with the observed true bearings to check the accuracy of the traverse.

1.24 CONSECUTIVE CO-ORDINATES: LATITUDE AND DEPARTURE

The theodolite traverse is not plotted according to interior angles or bearings. It is plotted by computing the latitudes and departures of the points (consecutive co-ordinates) and then finding the independent coordinates of the points.

The latitude of a line is the distance measured parallel to the North-South line and the departure of a line is measured parallel to the East-West line.

The latitude of a survey line may be defined as its co-ordinate length measured parallel to an assumed meridian direction *(i.e.* true north or magnetic north or any other reference direction-Parallel to N-S axis). In other words, latitude is the projection of a survey line on the North-South line.

$$\text{Latitude, L} = l\cos\theta$$

Where, l and θ are the lengths and R.B. of survey lines.

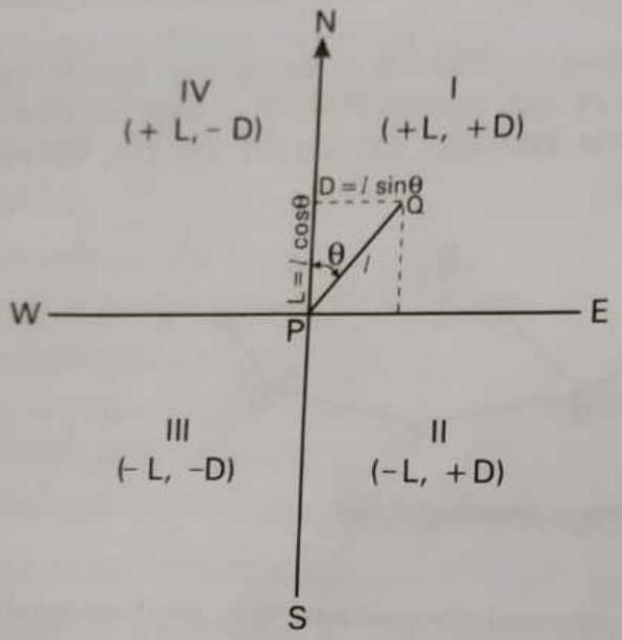
When the latitude of a line is measured northward (or upward) it is termed as 'Northing' and is positive.

When the latitude of a line is measured southward (downward) it is termed as 'Southing' and is negative.

The departure of survey line may be defined as its co-ordinate length measured at right angles to the meridian direction. (i.e., parallel to E-W axis). In other words, departure is the projection of a survey line on the East-West line.

$$\text{Departure, } D = l \sin\theta$$

When the departure of a line is measured eastward, it is termed as 'Easting' and is positive. When the departure of a line is measured westward it is termed as 'Westing' and is negative. The sign of Latitude and Departure will depend upon the quadrant in which a line lies.



l = Length of PQ

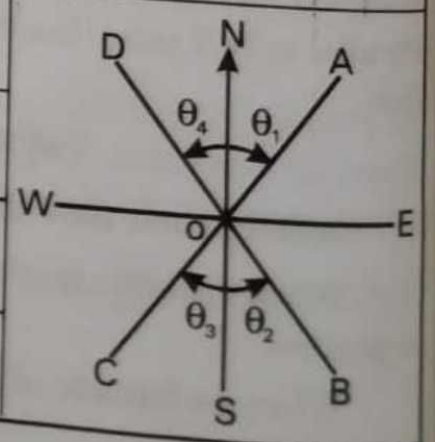
Latitude of line PQ = $l \cos\theta$

Departure of line PQ = $l \sin\theta$

Fig. 1.16 : Latitude and Depature

Note: While calculating latitude and departure by using calculator, if θ is in W.C.B, signs are displayed by calculator itself. If θ is in Q.B , the sign of latitude and departure should be properly considered depending upon quadrant.Table-1.1 gives the signs of latitudes and departures of lines OA, OB, OC and OD.

Table-1.1

Line	Quadrant	W.C.B	R.B	Signs of		Reference sketch
				Latitude	Departure	
OA	I	0^0 to 90^0	$N\,\theta_1\,E$	+	+	
OB	II	90^0 to 180^0	$S\,\theta_2\,E$	−	+	
OC	III	180^0 To 270^0	$S\,\theta_3\,W$	−	−	
OD	IV	270^0 to 360^0	$N\,\theta_4\,W$	+	−	

In a closed traverse, the algebraic sum of the latitudes and that of the departures should be zero.

In other words, the sum of all the north latitudes (Northings) should be equal to that of the south latitudes (Southings) and the sum of all the east departures (Eastings) should be equal to that of the west departures (Westings). The difference should be adjusted based on some agreed procedure characterised by a particular rule.

Consecutive co-ordinates of a station is designated by its departure and latitude from its previous station as origin or Latitude and departure co-ordinates of any point with reference to the preceeding point are known as "Consecutive co-ordinates". They are also called 'Dependent co-ordinates'.

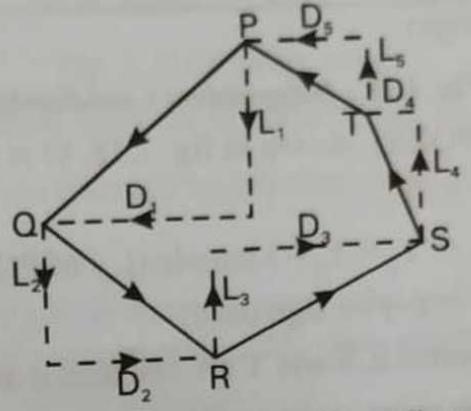


Fig. 1.17 : Consecutive Co-ordinates

Refer fig.1.17. Latitude and departure of lines PQ, QR, RS, ST and TP are shown in dotted lines, which are known as co-ordinates of PQ, QR, RS, ST and TP.

As per the definition of Consecutive co-ordinates, to find the Consecutive co-ordinate of P, the proceeding point T is taken as reference point. Hence, consecutive co-ordinates of P is the latitude and departure of TP.

i.e., Consecutive co-ordinates of P = Latitude and departure w.r.t. preceding point T.

Consecutive co-ordinates of P are i.e., latitude and departure of TP.

Similarly, Consecutive co-ordinates of Q, R, S and T can be calculated as below:

Consecutive co-ordinates of Q = Latitude and departure of PQ.

Consecutive co-ordinates of R = Latitude and departure of QR.

Consecutive co-ordinates of S = Latitude and departure of RS.

Consecutive co-ordinates of T = Latitude and departure of ST.

1.25 INDEPENDENT CO-ORDINATES

The co-ordinates of a point with reference to a common origin are known as "Independent co-ordinates". These co-ordinates are also called "Total latitude" and "Total departure" of that point.

The total latitude (Y) of any point is obtained by algebraically adding the latitudes of the lines between that point and the origin. Thus, total latitude of any point = Y co-ordinate of origin + algebraic sum of latitudes of all the lines up to that point.

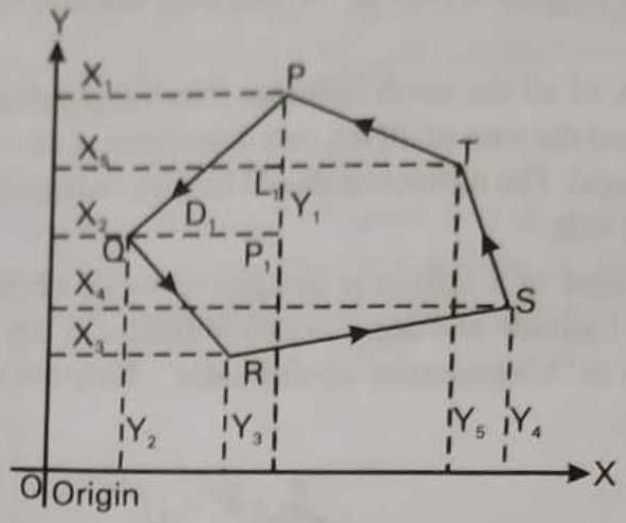


Fig. 1.18 : Independent Coordinates

Consider a closed traverse PQRST shown in fig .1.18, O is the origin.

Total latitude of P = Y_1

Total latitude of Q = $Y_2 = Y_1 -$ Latitude (L_1) of PQ [$\because$ Latitude of PQ is southing (–) ve sign used]

Similarly total latitudes of points R,S and T are calculated as Y_3, Y_4 and Y_5 respectively.

The total departure (X) of any point is obtained by algebraically adding the departures of the lines between that point and the origin.

Thus, total departure of any point = X Co-ordinate of origin + algebraic sum of departures of all the lines up to that point.

Refer Fig. 1.18

Total departure of P = X_1

Total departure of Q = $X_2 = X_1 -$ departure (D_1) of PQ

($\because$ Departure of PQ is Westing (–) ve sign used)

Similarly total departures of R, S and T are calculated as X_3, X_4 and X_5.

The origin is so chosen that the whole of the traverse lies in the north-east quadrant to have positive independent co- ordinates for all the points. To have the co-ordinates of all the points as positive, suitable arbitrary values are assigned to the origin.

Total latitude (or departure) of end point of a traverse = total latitudes (or departures) of first point of traverse plus the algebraic sum of all the latitudes (or departures).

1.26 CLOSING ERROR

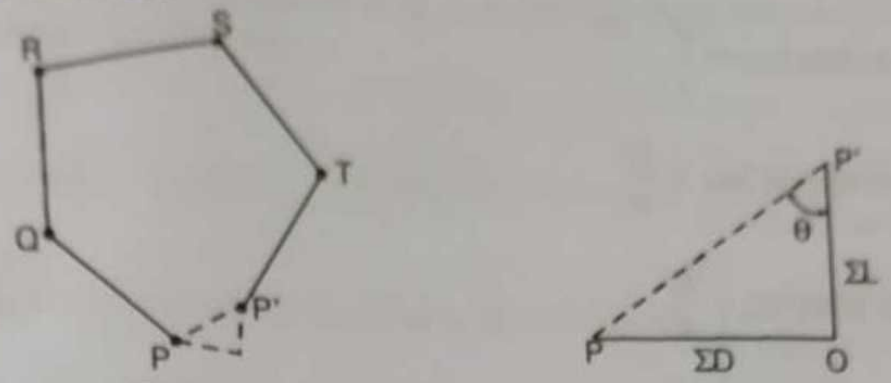


Fig. (a) *Fig. (b)*

PP' → closing error

Fig. 1.19 : Closing error

If the survey work of a closed traverse is correct, the conditions of the closed traverse $\sum Latitude = 0$ and $\sum Departure = 0$ should be satisfied. If the above conditions are not satisfied, there is an error of closure.

If the traverse is plotted using field measurements, the end point will not coincide with the starting point due to errors in field measurements. The distance by which the end point falls short to coincide with starting point is called 'closing error'.

In fig. 1.19, PP' is the closing error.

$$\text{Closing error, } e = PP' = \sqrt{(\sum Departure)^2 + (\sum Latitude)^2}$$

$$\text{Direction of closing error} = \tan\theta = \frac{\sum Departure}{\sum Latitude}$$

Where, θ is the Reduced bearing of closing error.

$$\text{Relative error of closure} = \frac{\text{Error of closure}}{\text{Perimeter of traverse}} = \frac{e}{p} = \frac{1}{p/e}$$

1.27 ADJUSTMENT OF ERRORS IN ANGLES AND BEARINGS

Adjustment of angular error

The error in the sum of interior or exterior angles is distributed equally to each angle of the traverse if it is within the permissible limit.

Adjustment of errors in Bearings

Let 'e' be the closing error in fore and back bearings of the same line (say last line) of a closed traverse. Let n be the number of sides.

Then, corrections to bearings of various lines are as follows:

Correction to first line $= \frac{e}{n}$

Correction to second line $= \frac{2e}{n}$

Correction to third line $= \frac{3e}{n}$

Correction to fourth line $= \frac{4e}{n}$

Correction to last line $= \frac{Ne}{n} = e$

1.28 BALANCING THE TRAVERSE

The process of applying correction to latitudes and departures so as to make algebraic sum of latitudes and departures are equal to zero is known as 'Balancing the traverse'. By applying corrections, the traverse is made to close geometrically.

The sign of correction is opposite to that of error. This process is applied for a closed traverse only. The following methods are commonly used.

1. Bowditch's rule
2. Transit rule
3. Graphical method

Bowditch's rule

Bowditch's rule also called 'Compass rule' is generally used for adjusting a traverse in which the angles and distances are measured with equal precision. In this rule, the total error in latitude and departure is distributed in proportion to the length of the sides.

Thus,

$$\text{Correction to latitude of any line} = \text{Total error in latitude} \times \frac{\text{Length of that line}}{\text{Perimeter of the traverse}}$$

$$\text{Correction to departure of any line} = \text{Total error in departure} \times \frac{\text{Length of that line}}{\text{Perimeter of the traverse}}$$

Transit rule

Transit rule is used to balance a traverse in which the angular measurements are more precise than the linear measurements.

According to this rule, the total error in latitude and departure is distributed in proportion to the latitude and departure of the sides.

Thus,

$$\text{Correction to latitude of any line} = \text{Total error in latitude} \times \frac{\text{Latitude of that line}}{\text{Arithmetic sum of latitudes}}$$

$$\text{Correction to departure of any line} = \text{Total error in departure} \times \frac{\text{departure of that line}}{\text{Arithmetic sum of departures}}$$

The Bowditch rule is particularly useful for a compass traverse where the angles are susceptible to considerable error whereas the transit rule is more suited for a theodolite traverse where the possibility of error is more in the linear rather than in the angular measurements.

1.29 AREA OF A CLOSED TRAVERSE

Area of closed traverse can be computed by the following methods:

1. Independent co-ordinates method.
2. Latitude and double meridian distance method.
3. Departures and total latitudes method.

1. Independent co-ordinates method

Let PQRST be a closed traverse whose area is to be computed by Independent co-ordinates. (Ref. fig.1.20).

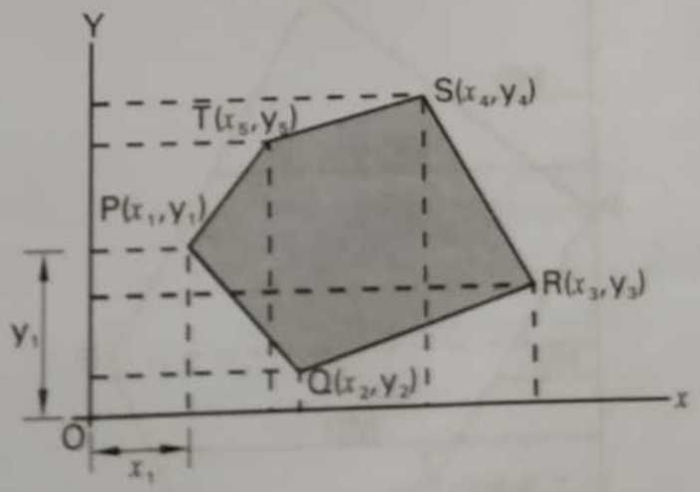


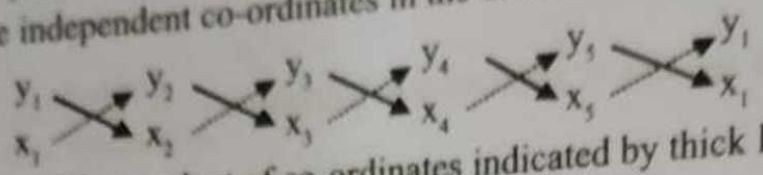
Fig. 1.20 : Area from Coordinates

Following procedure is used to compute the area.

1. Calculate the latitude and departure of lines PQ, QR, RS, ST and TP from their lengths and reduced bearings.
2. Calculate the Consecutive co-ordinates of points, P, Q, R, S and T.
3. Calculate the Independent co-ordinates of points P, Q, R, S and T.

Assume suitable arbitrary values for the origin so that all the points have positive values.

4. Let y_1, y_2, y_3, y_4 and y_5 be Y co-ordinates of P, Q, R, S and T respectively.
5. Let x_1, x_2, x_3, x_4 and x_5 be X co-ordinates of P, Q, R, S and T respectively.
6. Arrange the independent co-ordinates in the determinant form in two rows as shown below



7. Find the sum of the product of co-ordinates indicated by thick lines.

   i.e., $\sum P = y_1x_2 + y_2x_3 + y_3x_4 + y_4x_5 + y_5x_1$

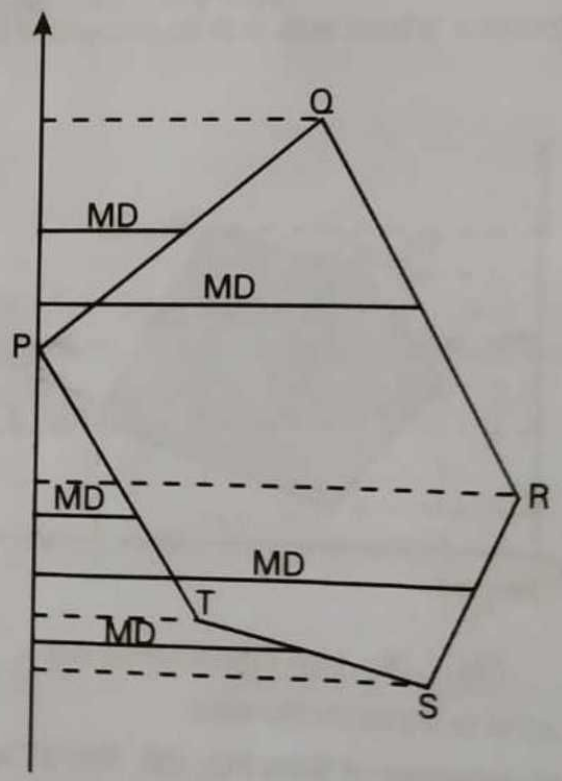
8. Find the sum of the product of co-ordinates indicated by dotted lines

   i.e., $\sum Q = x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 + x_5y_1$

9. Then, twice area, 2A = Difference between $\sum P$ and $\sum Q = \sum P - \sum Q$

10. Therefore, Area of the traverse, $A = \dfrac{\sum P - \sum Q}{2}$ Sq units.

2. Latitude and Double Meridian Distance Method (DMD)



MD - Meridian distance

Fig. 1.21 : Latitude and DMD Method

1.30 OMITTED MEASUREMENTS

As a rule traverses are always closed, either onto themselves or between known points so that an estimate of accuracy and precision can be obtained, as well as a check on our fieldwork.

The length and bearing of each line of a traverse should be completely surveyed to

a. Plot the traverse,

b. Compute the area of the traverse,

c. Check the accuracy of linear and angular measurements made,

d. Balance the traverse.

Sometimes, it may not be possible or practicable to measure all linear and angular measurements in the field due to intervening obstacle or accidental omission, etc.

The measurements which could not be measured or missed to measure are called omitted measurements.

The missing quantities or omitted measurements can be determined using the principle of a closed traverse that is,

i) Algebraic sum of latitudes is equal to zero and

ii) Algebraic sum of departures is equal to zero.

$$\sum L = 0 \text{ and } \sum D = 0 \text{ (or)}$$

$$l_1 \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 + \ldots\ldots = 0 - - - - - 1$$

$$l_1 \sin\theta_1 + l_2 \sin\theta_2 + l_3 \sin\theta_3 + \ldots\ldots = 0 - - - - - 2$$

Where $l_1, l_2, l_3 \ldots\ldots$ etc., are the lengths of traverse lines and

$\theta_1, \theta_2, \theta_3 \ldots\ldots$ etc., are the corresponding reduced bearings.

The missing quantities can be determined from the above two conditions, assuming that, there are no errors in the observed measurements.

Only two omitted measurements of a traverse can be determined at a time as there are only two conditions available.

The omitted elements of a traverse polygon that can be solved for are as follows:

1. Bearing and Distance of One Line.
2. Bearing of One Line, Distance of another.
3. Distance of two Lines.
4. Bearing of two Lines.

1.31 WORKED EXAMPLES

- Deflection angle calculation.
- Computing Latitude and departure.
- Closing error, Bowditch & Transit rule.
- Area of traverse.
- Omitted measurements.

1.31.1 DEFLECTION ANGLE CALCULATION

1. An open traverse is run from A to G & deflection angles are as follows:

Station	Deflection angle
B	$32^0 16'$ (L)
C	$18^0 34'$ (R)
D	$22^0 12'$ (L)
E	$42^0 24'$ (R)
F	$52^0 42'$ (R)

Bearing of the first line is $110^0 6'$, calculate the bearing of all other lines.

Solution:

Bearing of AB = $110^0 6'$ (Given)

Bearing of BC = $110^0 6' - 32^0 16' = 77^0 50'$

Bearing of CD = $77^0 50' + 18^0 34' = 96^0 24'$

Bearing of DE = $96^0 24' - 22^0 12' = 74^0 12'$

Bearing of EF = $74^0 12' + 42^0 24' = 116^0 36'$

Bearing of FG = $116^0 36' + 52^0 42' = 169^0 18'$

2. Reduce the following interior angles of closed traverse ABCDEF to deflection angles.

$\angle A - 68^0 30', \angle B - 208^0 26', \angle C - 83^0 14', \angle D - 62^0 10', \angle E - 214^0 20', \angle F - 83^0 20'$

Solution:

Deflection angle at A = $180^0 - 68^0 30' = 111^0 30'$ R

Deflection angle at B = $208^0 26' - 180^0 = 28^0 26'$ L

Deflection angle at C = $180^0 - 83^0 14' = 96^0 46'$ R

Deflection angle at D = $180^0 - 62^0 10' = 117^0 50'$ R

Deflection angle at E = $214^0 20' - 180^0 = 34^0 20'$ L

Deflection angle at F = $180^0 - 83^0 20' = 96^0 40'$ R

Sum of deflection angles to Right = $111^0 30' + 96^0 46' + 117^0 50' + 96^0 40' = 422^0 46'$

Sum of deflection angles to Left = $28^0 26' + 34^0 20' = 62^0 46'$

Check : For closed traverse algebraic sum deflection angle must be equal to 360^0

$422^0 46' - 62^0 46' = 360^0$

3. **The following interior angles of closed traverse ABCDEF. Calculate the deflection angles.**

$\angle A - 60^0 40', \angle B - 201^0 38', \angle C - 93^0 19', \angle D - 69^0 48', \angle E - 210^0 13', \angle F - 84^0 22'$

Solution:

Deflection angle at A = $180^0 - 60^0 40' = 119^0 40'$ R

Deflection angle at B = $201^0 38' - 180^0 = 21^0 38'$ L

Deflection angle at C = $180^0 - 93^0 19' = 86^0 41'$ R

Deflection angle at D = $180^0 - 69^0 48' = 110^0 12'$ R

Deflection angle at E = $210^0 13' - 180^0 = 30^0 13'$ L

Deflection angle at F = $180^0 - 84^0 22' = 95^0 38'$ R

Sum of deflection angles to Right = $95^0 38' + 110^0 12' + 86^0 41' + 119^0 20' = 411^0 51'$

Sum of deflection angles to Left = $30^0 13' + 21^0 38' = 51^0 51'$

Check: For closed traverse algebraic sum deflection angle must be equal to 360^0

$411^0 51' - 51^0 51' = 360^0$

1.30.2 COMPUTING LATITUDE AND DEPARTURE

1. **Following are the bearings of a traverse. Calculate its latitude and departure of a line**

Line	Length in m	Bearing
AB	202.1	$N30^0 31'E$
BC	101.6	$N42^0 20'W$
CD	98.5	$S60^0 E$
DE	75.5	$S25^0 25'W$

Solution:

Latitude = $l \cos\theta$

Latitude of AB = $202.1 \cos 30^0 31' = +174.10m$ (+ve – 1st quadrant)

Latitude of BC = $101.6 \cos 42^0 20' = +75.1m$ (+ve – 4th quadrant)

Latitude of CD = $98.5\cos 60^0 = -49.25m$ (-ve – 2nd quadrant)

Latitude of DE = $75.5\cos 25^0 25' = -68.19m$ (-ve- 3rd quadrant)

Departure = $l\sin\theta$

Departure of AB = $202.1\sin 30^0 31' = +102.62m$

Departure of BC = $101.6\sin 42^0 20' = -68.42m$

Departure of CD = $98.5\sin 60^0 = +85.30m$

Departure DE = $75.5\sin 25^0 25' = -32.40m$

2. Following are the bearings of a traverse. Calculate its latitude and departure of a line

Line	Length in m	Bearing
AB	70	83^0
BC	110	120^0
CD	90	190^0
DE	343	255^0
EA	212	315^0

Solution:

Latitude = $l\cos\theta$

Latitude of AB = $70\cos 83^0 = +8.53m$

Latitude of BC = $110\cos 120^0 = -55m$

Latitude of CD = $90\cos 190^0 = -88.63m$

Latitude of DE = $343\cos 255^0 = -88.77m$

Latitude of EA = $212\cos 315^0 = +149.90m$

Departure = $l\sin\theta$

Departure of AB = $70\sin 83^0 = +69.47m$

Departure of BC = $110\sin 120^0 = +95.26m$

Departure of CD = $90\sin 190^0 = -15.62m$

Departure of DE = $343\sin 255^0 = -331.31m$

Departure of EA = $212\sin 315^0 = -149.90m$

3. **Following are the readings observed in traversing with a theodolite:**

Line	Bearings	Length in m
AB	$343^0 50'$	370
BC	$87^0 50'$	540
CD	$172^0 40'$	340
DA	$265^0 10'$	490

Calculate the latitude and departure of sides of the traverse ABCDA.

Solution:

Latitude = $l\cos\theta$

Latitude of AB = $370\cos 343^0 50' = +355.36m$

Latitude of BC = $540\cos 87^0 50' = +20.41m$

Latitude of CD = $340\cos 172^0 40' = -337.21m$

Latitude of DA = $490\cos 265^0 10' = -41.28m$

Departure = $l\sin\theta$

Departure of AB = $370\sin 343^0 50' = -103.02m$

Departure of BC = $540\sin 87^0 50' = +539.61m$

Departure of CD = $340\sin 172^0 40' = +43.39m$

Departure of DA = $490\sin 265^0 10' = -488.26m$

4. **The coordinates of two points C & D are as follows. Find the length and bearing of C, D.**

Point	Co-ordinates	
	N	E
C	980	820
D	1192	567.3

Solution:

Latitude of CD = Difference between north coordinates of points D & C = 1192 – 980 = + 212m

Departure of CD = Difference between east coordinates of D & C = 567.3 – 820 = –252.7m

Length of CD = $\sqrt{Latitude^2 + Departure^2} = \sqrt{212^2 + 252.7^2} = 329.85m$

$$\theta = \tan^{-1}\frac{Departure}{Latitude} = \tan^{-1}\frac{252.7}{212} = 50^0 0' 20''$$

since, latitude + ve and departure - ve, θ lies in IV qudrant (i.e., NW)

$\theta = N50^0 0' 20'' W$

$W.C.B = 360^0 - 50^0 0' 20'' = 309^0 59' 40''$

5. The coordinates of two points A & B are as follows:

Point	Co-ordinates	
	N	E
A	481.5	324.2
B	607.6	73.4

Find the length and bearing of line AB.

Solution:

Latitude of AB = 607.5 – 481.5 = + 126m

Departure of AB = 73.4 – 324.2 = – 250.8m

Length of AB $= \sqrt{Latitude^2 + Departure^2} = \sqrt{126^2 + 250.8^2} = 280.67m$

$$\theta = \tan^{-1}\frac{Departure}{Latitude} = \tan^{-1}\frac{250.8}{126} = 63^0 19' 32''$$

since, latitude + ve and departure - ve, θ lies in IV qudrant (i.e., NW)

$\theta = N63^0 19' 32'' W$

$W.C.B = 360^0 - 63^0 19' 32'' = 296^0 40' 28''$

6. A man travels from 'A' towards west and reaches the point 'B'. The distance between A & B = 140m. Calculate the latitude and departure of the line AB.

Solution:

W.C.B of AB = 270^0

Latitude of AB = $l\cos\theta = 140\cos 270^0 = 0$

Departure of AB = $l\sin\theta = 140\sin 270^0 = -140m$

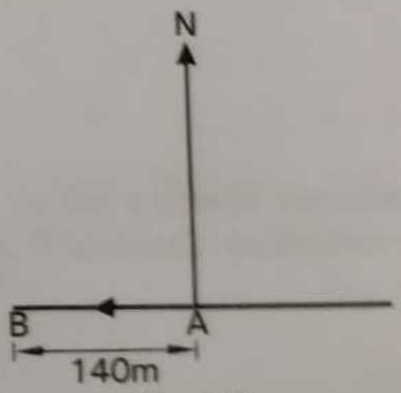


Fig. 1.23

7. The bearings of PQ and QR are 19°22' and 60°20' respectively. The Co-ordinates of the ends of P and R are

Point	Co-ordinates	
	N	E
P	200.00	300.00
R	1332.80	1157.20

Find the length of PQ and QR.

Solution:

Latitude of PR = 1332.8 – 200 = 1132.8m

Departure of PR = 1157.2 – 300 = 857.2m

Length of PR = $\sqrt{Latitude^2 + Departure^2} = \sqrt{1132.8^2 + 857.2^2} = 1420.57m$

Magnetic bearing of PR = $\theta = \tan^{-1}\left(\frac{Departure}{Latitude}\right) = \tan^{-1}\left(\frac{857.2}{1132.8}\right) = 37^0 6'55''$

$\theta = N37^0 6'55'' E$

$W.C.B = 37^0 6'55''$

In triangle PQR

Bearing of QP = +180° + 19°22' = 199°22'

Angle Q = Bearing of QP – Bearing of QR = 199°22' – 60°20' = 139°2'

Angle P = Bearing of PR – Bearing of PQ = 37°6'55'' – 19°22'' = 17°44'55''

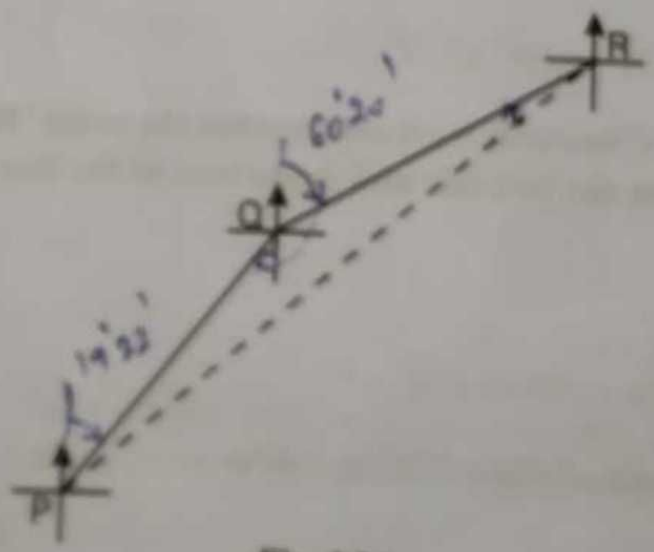


Fig. 1.24

Angle R = 180° – (139°2' + 17°44'55") = 23°13'5"

Applying sine rule in triangle PQR

$$\frac{PR}{Sin\,Q} = \frac{RQ}{Sin\,P}$$

$$RQ = \frac{PR \times \sin P}{\sin Q} = \frac{1420.57 \times \sin 17^0 44'55''}{\sin 139^0 2'} = 660.53m$$

$$\frac{PR}{Sin\,Q} = \frac{PQ}{Sin\,R}$$

$$PQ = \frac{PR \times \sin R}{\sin Q} = \frac{1420.57 \times \sin 23^0 13'5''}{\sin 139^0 2'} = 854.2m$$

8. The coordinates of two points A & B are as follows:

Point	Co-ordinates	
	N	E
A	982.5	825.2
B	1198.6	576.4

Find the length and bearing of line AB.

Solution:

Latitude of AB = 1198.6 – 982.5 = + 216.1m

Departure of AB = 576.4 – 825.2 = – 248.8m

$$\text{Length of AB} = \sqrt{Latitude^2 + Departure^2} = \sqrt{216.1^2 + 248.8^2} = 329.54m$$

$$\theta = \tan^{-1}\frac{Departure}{Latitude} = \tan^{-1}\frac{248.8}{216.1} = 49^0 1'25''$$

$$\theta = N49^0 1' 25'' W$$

$$W.C.B = 360^0 - 49^0 1' 25'' = 310^0 58' 35''$$

9. Co-ordinates of line AB are -658.7, +468.2 with respect to the magnetic meridian. Calculate the co-ordinates with respect to true meridian, if magnetic declination is 10°12'E.

The results are tabulated as below: Following are lat & dep of sides of traverse ABCD. Cal consecutive co-ord & Independent co-ord.?

Line	Latitude	Departure	Point	Consecutive Co-ordinates		Independent Co-ordinates	
				Latitude Y	Departure X	Y	X
AB	+107.40	+62.00	A	+ 93.10	-119.90	400.00	400.00
BC	-122.60	+ 102.90	B	+107.40	+ 62.00	507.40	462.00
CD	-77.90	-45.00	C	-122.60	+102.90	384.80	564.90
DA	+93.10	-119.90	D	- 77.90	- 45.00	306.90	519.90
			A	+ 93.10	-119.90	400.00	400.00

1.30.3 PROBLEMS ON CLOSING ERROR, BOWDITCH & TRANSIT RULE

1. **In a closed traverse $\sum Latitude = 1.39$ & $\sum$ Departure= -2.17 calculate the length and direction of the closing error & also relative error of closure, if P=1500m.**

Solution:

$$e = \sqrt{(\sum Latitude)^2 + (\sum Departure)^2} = \sqrt{(1.39)^2 + (-2.17)^2} = 2.58m$$

$$\tan\theta = \frac{\sum Departure}{\sum Latitude} = \frac{2.17}{1.39} = 1.56$$

$$\theta = N57^0 21' 29'' W$$

$$\text{Relative error of closure} = \frac{1}{P/e} = \frac{1}{1500/2.58} = \frac{1}{581.39} \text{ say 1 in 582}$$

2. Following are the length and bearings of sides of closed traverse ABCD. The bearings given are with respect to magnetic meridian & declination is 5°30'W. Convert the bearings to true bearing and find relative error of closure.

Line	Length in m	Bearing
AB	470	343°52'
BC	635	87°50'
CD	1130	172°40'
DA	563	265°12'

Solution:

Line	Length in m	Bearing	True bearing of line	Latitude $= l\cos\theta$	Departure $= l\sin\theta$
AB	470	343°52'	338°22'	+436.89	-173.27
BC	635	87°50'	82°20'	+84.715	+629.32
CD	1130	172°40'	167°10'	-1101.77	+250.99
DA	563	265°12'	259°42'	-100.66	-553.92
			$\sum =$	-680.83	+153.12

Note: True bearing of line = Magnetic bearing – Declination for west

$$e = \sqrt{(\sum Latitude)^2 + (\sum Departure)^2} = \sqrt{(-680.83)^2 + (153.12)^2} = 697.84m$$

$$Perimeter\ P = 470 + 635 + 1130 + 563 = 2798m$$

$$\text{Relative error of closure} = \frac{1}{P/e} = \frac{1}{2798/697.84} = \frac{1}{4}$$

3. The following are the lengths and bearings of the sides of traverse ABCD. Find the closing error and its direction.

Line	Length in m	Bearings
AB	470	343°52'
BC	635	172°40'
CD	430	172°40'
DA	563	265°12'

Solution:

$$Latitude = l\cos\theta$$

$$\sum Latitude = 470\cos 343^0 52' + 635\cos 172^0 40' + 430\cos 172^0 40' + 563\cos 265^0 12'$$

$$= 451.49 - 629.80 - 426.48 - 47.11$$

$$\sum Latitude = -651.9$$

$Departure = l\sin\theta$

$$\sum Departure = 470\sin 343^0 52' + 635\sin 172^0 40' + 430\sin 172^0 40' + 563\sin 265^0 12'$$

$$= -130.6 + 81.05 + 54.88 - 561.02$$

$$\sum Departure = -555.69$$

$$e = \sqrt{(\sum Latitude)^2 + (\sum Departure)^2} = \sqrt{(-651.9)^2 + (-555.69)^2} = 856.61m$$

$$\theta = \tan^{-1}\frac{\sum Departure}{\sum Latitude} = \tan^{-1}\frac{555.69}{651.9} = 40^0 26'42''$$

$$\theta = S40^0 26'42''W$$

4. Adjust the following traverse by Bowditch Rule

Line	Length in m	Bearing
AB	**89.31**	**$45^0 10'$**
BC	**219.76**	**$72^0 05'$**
CD	**151.18**	**$161^0 52'$**
DE	**159.10**	**$228^0 43'$**
EA	**232.26**	**$300^0 42'$**

Solution:

Line	Length In m	Bearing	Latitude	Departure	Correction to latitude	Correction to departure	Corrected latitude	Corrected departure
AB	89.31	$45^0 10'$	+62.97	+63.34	-0.055	-0.023	+62.915	+63.317
BC	219.76	$72^0 05'$	+67.61	+209.10	-0.134	-0.057	+67.476	+209.043
CD	151.18	$161^0 52'$	-143.67	+47.05	-0.092	-0.039	-143.762	+47.011
DE	159.10	$228^0 43'$	-104.97	-119.56	-0.097	-0.041	-105.067	-119.601
EA	232.26	$300^0 42'$	+118.58	-199.71	-0.142	-0.060	+118.438	-199.770
Total=	**851.61**		**+0.52**	**+0.22**	**-0.52**	**-0.22**	**0**	**0**

Perimeter = 851.61m

Total error in latitude = +0.52

Total error in departure = +0.22

Correction to latitude or departure of any line =

$$\text{Total error in latitude or departure} \times \frac{\text{Length of that line}}{\text{Perimeter of the traverse}}$$

Correction to latitude of line

$$AB = -0.52 \times \frac{89.31}{851.61} = -0.055$$

$$BC = -0.52 \times \frac{219.76}{851.61} = -0.134$$

$$CD = -0.52 \times \frac{151.18}{851.61} = -0.092$$

$$DE = -0.52 \times \frac{159.10}{851.61} = -0.097$$

$$EA = -0.52 \times \frac{232.26}{851.61} = -0.142$$

Correction departure of line

$$AB = -0.22 \times \frac{89.31}{851.61} = -0.023$$

$$BC = -0.22 \times \frac{219.76}{851.61} = -0.057$$

$$CD = -0.22 \times \frac{151.18}{851.61} = -0.039$$

$$DE = -0.22 \times \frac{159.10}{851.61} = -0.041$$

$$EA = -0.22 \times \frac{232.26}{851.61} = -0.060$$

5. Calculate the error of closure and adjust the following traverse by using transit rule.

Line	Length in m	Bearing
PQ	130	S88^{0}E
QR	158	S6^{0}E
RS	145	S40^{0}W
ST	308	N81^{0}W
TP	337	N48^{0}E

Solution:

Line	Length In m	Bearing	Latitude	Departure	Correction to latitude	Correction to departure	Corrected latitude	Corrected departure
PQ	130	S88^{0}E	-4.54	+129.92	-0.008	+0.087	-4.548	130.007
QR	158	S6^{0}E	-157.13	+16.52	-0.268	+0.011	-157.398	16.531
RS	145	S40^{0}W	-111.08	-93.20	-0.189	+0.062	-111.269	-93.138
ST	308	N81^{0}W	+48.18	-304.21	-0.082	+0.203	48.098	-304.007
TP	337	N48^{0}E	+225.50	+250.44	-0.383	+0.167	225.117	250.607
		Total=	**+0.93**	**-0.53**	**-0.93**	**+0.53**	**0**	**0**

Error of closure, $e = \sqrt{(\sum Latitude)^2 + (\sum Departure)^2} = \sqrt{(0.93)^2 + (-0.53)^2} = 1.07\,m$

Arithmetic sum of latitudes = 4.54 + 157.13 + 111.08 + 48.18 + 225.5 = 546.43m

Arithmetic sum of departure = 129.92 + 16.52 + 93.2 + 304.21 + 250.44 = 794.29m

Correction to latitude or departure of any line =

$$\text{Total error in latitude or departure} \times \frac{\text{Latitude or departure of that line}}{\text{Arithmetic sum of latitudes or departures}}$$

Correction to latitude of line

$$PQ = -0.93 \times \frac{4.54}{546.43} = -0.008$$

$$QR = -0.93 \times \frac{157.13}{546.43} = -0.268$$

$$RS = -0.93 \times \frac{111.88}{546.43} = -0.189$$

$$ST = -0.93 \times \frac{48.18}{546.43} = -0.082$$

$$TP = -0.93 \times \frac{225.50}{546.43} = -0.383$$

Correction departure of line

$$PQ = +0.53 \times \frac{129.92}{794.29} = +0.087$$

$$QR = +0.53 \times \frac{16.52}{794.29} = +0.011$$

$$RS = +0.53 \times \frac{93.20}{794.29} = +0.062$$

$$ST = +0.53 \times \frac{304.21}{794.29} = +0.203$$

$$TP = +0.53 \times \frac{250.44}{794.29} = +0.167$$

6. **Calculate the error of closure and adjust the following traverse by using transit rule (units in m).**

Line	Latitude	Departure
PQ	123.35	35.68
QR	93.82	205.86
RS	-177.44	70.11
SP	-39.21	-312.25

Solution:

Line	Latitude	Departure	Correction to latitude	Correction to departure	Corrected latitude	Corrected departure
PQ	123.35	35.68	-0.148	0.034	123.202	35.714
QR	93.82	205.86	-0.112	0.198	93.708	206.058
RS	-177.44	70.11	-0.213	0.067	-177.653	70.177
SP	-39.21	-312.25	-0.047	0.300	-39.257	-311.950
Total=	**0.52**	**-0.6**	**-0.520**	**0.600**	**0.000**	**0.000**

Error of closure, $e = \sqrt{(\sum Latitude)^2 + (\sum Departure)^2} = \sqrt{(0.52)^2 + (-0.60)^2} = 0.793$ m

Arithmetic sum of latitudes = 123.35 + 93.82 + 177.44 + 39.21 = 433.82

Arithmetic sum of departure = 35.68 + 205.86 + 70.11 + 312.25 = 623.9

Specimen calculation:

Correction to latitude or departure of any line =

$$\text{Total error in latitude or departure} \times \frac{\text{Latitude or departure of that line}}{\text{Arithmetic sum of latitudes or departures}}$$

$$\text{Correction to latitude of line PQ} = -0.52 \times \frac{123.35}{433.82} = -0.148$$

$$\text{Correction departure of line PQ} = +0.60 \times \frac{35.68}{623.9} = +0.034$$

7. **The following table gives the details of the closed traverse ABCD. Find the**
   a) **Closing error and relative error of closure.**
   b) **Adjust the traverse by Bowditch rule.**
   c) **Independent coordinates of all the points.**

Station	N	S	E	W	Length
AB	131.07			51.99	141
BC	25.41		188.82		190.5
CD		125.79	28.656		129
DA		30.15		165.87	168

Solution:

$$\sum Latitude = 131.07 + 25.41 - 125.79 - 30.15 = 0.54$$

$$\sum Departure = -51.99 + 188.82 + 28.656 - 165.87 = -0.384$$

Closing error

$$e = \sqrt{(\sum Latitude)^2 + (\sum Departure)^2} = \sqrt{(0.54)^2 + (-0.384)^2} = 0.662m$$

$$\theta = \tan^{-1}\frac{\sum Departure}{\sum Latitude} = \tan^{-1}\frac{0.384}{0.54} = 35^0 25'1''$$

$$\theta = N35^0 25'1''W$$

$$\text{Perimeter} = 141 + 190.5 + 129 + 168 = 628.5m$$

$$\text{Relative error of closure} = \frac{1}{P/e} = \frac{1}{628.5/0.662} = \frac{1}{949.39} \text{ say 1 in 950}$$

Bowditch rule :

Line	Length	Latitude	Departure	Correction to latitude	Correction to departure	Corrected latitude	Corrected departure
AB	141.0	131.07	-51.99	-0.121	0.086	130.949	-51.904
BC	190.5	25.41	188.82	-0.164	0.116	25.246	188.936
CD	129.0	-125.79	28.656	-0.111	0.079	-125.901	28.735
DA	168.0	-30.15	-165.87	-0.144	0.103	-30.294	-165.767
Total=	**628.5**	**+0.54**	**-0.384**	**-0.54**	**+0.384**	**0**	**0**

Perimeter = 628.5m

Total error in latitude = + 0.54

Total error in departure = – 0.384

Specimen calculation:

Correction to latitude or departure of any line =

$$\text{Total error in latitude or departure} \times \frac{\text{Length of that line}}{\text{Perimeter of the traverse}}$$

$$\text{Correction to latitude of line AB} = -0.54 \times \frac{141}{628.5} = -0.121$$

$$\text{Correction departure of line AB} = +0.384 \times \frac{141}{628.5} = +0.086$$

Let the independent co-ordinates of A be (100,100)

Point	Total latitude	Total departure
A	100	100
B	100+130.949=230.949	100-51.904=48.096
C	230.949+25.246=256.195	48.096+188.936=237.032
D	256.195-125.901=130.294	237.032+28.735=265.767
A(Check)	130.294-30.294=100	265.767-165.767=100

8. Balance the given traverse in Bowditch rule & Transit rule. Calculate closing error and relative error of closure.

Line	Length (m)	Latitude(m)	Departure(m)
AB	200	-173.2	100
BC	130	0	130
CD	100	86.6	50
DE	250	250	0
EA	320	-154.9	-280

Solution:

Line	Length in m	Latitude	Departure	Bowditch rule		Transit rule	
				Correction to latitude	Corrected latitude	Correction to latitude	Corrected latitude
AB	200	-173.2	100	-1.70	-174.9	-2.21	-175.41
BC	130	0	130	-1.11	-1.11	0	0
CD	100	86.6	50	-0.85	85.75	-1.11	85.49
DE	250	250	0	-2.12	247.88	-3.20	246.80
EA	320	-154.9	-280	-2.72	-157.62	-1.98	-156.88
Total=	**1000**	**+8.5**	**0**	**-8.5**	**0**	**-8.50**	**00**

Perimeter, P = 1000m

Total error in latitude = +8.5

Total error in departure = 0

Closing error

$$e = \sqrt{(\sum Latitude)^2 + (\sum Departure)^2} = \sqrt{(8.5)^2 + (0)^2} = 8.5m$$

$$\theta = \tan^{-1}\frac{\sum Departure}{\sum Latitude} = \frac{0}{8.5} = 0\text{ (North)}$$

$$\text{Relative error of closure} = \frac{1}{P/e} = \frac{1}{1000/8.5} = \frac{1}{117.64}$$

Specimen calculation:

Bowditch rule:

Perimeter, P = 1000m

Correction to latitude of any line =

$$\text{Total error in latitude} \times \frac{\text{Length of that line}}{\text{Perimeter of the traverse}}$$

$$\text{Correction to latitude of line AB} = -8.5 \times \frac{200}{1000} = -1.70$$

Transit rule:

Arithmetic sum of latitudes = 173.2 + 0 + 86.6 + 250 + 154.9 = 664.70

Correction to latitude of any line =

$$\text{Total error in latitude} \times \frac{\text{Latitude of that line}}{\text{Arithmetic sum of latitudes}}$$

$$\text{Correction to latitude of line AB} = -8.5 \times \frac{173.2}{664.70} = -2.21$$

9. The following table gives the consecutive co-ordinates of points forming a closed traverse ABCDE.

Station	N	S	E	W
A	300.75			200.5
B	200.25		299.25	
C		299	199.75	
D		200		
E				300.5

1.31.4 AREA OF TRAVERSE

1. **Find the area of the closed traverse having the following data, by the coordinate method.**

Side	Latitude	Departure
AB	+225.5	+120.5
BC	-245.0	+210.0
CD	-150.5	-110.5
DA	+170.0	-220.0

Solution:

The consecutive coordinates are arranged in independent coordinate form as follows:

The independent coordinates of the most westerly station A are assumed to be (+200,+100)

Station	Side	Consecutive coordinates		Independent coordinates	
		Latitude	Departure	North (y)	East(x)
A				+200.0	+100.0
B	AB	+225.5	+120.5	+200.0+225.55 = +425.5	+100.0+120.5= +220.5
C	BC	-245.0	+210.0	+425.5-245.0= +180.5	+220.5+210.0= +430.5
D	CD	-150.5	-110.5	+180.5-150.5= +30.0	+430.5-110.5= +320.0
A	DA	+170.0	-220.0	+30.0+170.0= +200.0	+320.0-220.0=+100.0

Thus, the independent coordinates of all the stations become positive (i.e., they come to the first quadrant).

The coordinates are now arranged in determinant form as follows:

y_1	y_2	y_3	y_4	y_1
x_1	x_2	x_3	x_4	x_1
200	425.5	180.5	30.0	200
100	220.5	430.5	320.0	100

Sum of the product of co-ordinates indicated by thick lines.

i.e., $\sum P = y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1$

$$\sum P = 200\times220.5 + 425.5\times430.5 + 180.5\times320.0 + 30.0\times100 = 288037.75$$

Sum of the product of co-ordinates indicated by dotted lines

ie., $\sum Q = x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1$

The following table gives lat & dep of ABCD. Compute its area?

Solution:

Co-ordinate method

Origin is so chosen that all the points lie in North-East quadrant.

Let the co-ordinates of point be 200,100

Station	Side	Consecutive coordinates		Independent coordinates	
		Latitude	Departure	North (y)	East(x)
A				200.0	100.0
B	AB	214.8	124.0	200.0+214.8 =414.8	100+124 =224
C	BC	-245.10	205.7	414.8-245.10 =169.7	224+205.7 =429.7
D	CD	-155.90	-90.0	169.7-155.90 =13.80	429.7-90 =339.7
A (Check)	DA	186.20	-239.70	13.80+186.20 =200.0	339.7-239.7 =100.0

The coordinates are now arranged in determinant form as follows:

$$\begin{matrix} y_1 & & y_2 & & y_3 & & y_4 & & y_1 \\ x_1 & & x_2 & & x_3 & & x_4 & & x_1 \end{matrix}$$

$$\begin{matrix} 200 & & 414.8 & & 169.7 & & 13.8 & & 200 \\ 100 & & 224 & & 429.7 & & 339.7 & & 100 \end{matrix}$$

$$\sum P = y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1$$

$$\sum P = 200\times224 + 414.8\times429.7 + 169.7\times339.7 + 13.8\times100 = 282066.65$$

$$\sum Q = x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1$$

$$\sum Q = 100\times414.8 + 224\times169.7 + 429.7\times13.8 + 339.7\times200 = 153362.66$$

$$\text{Area of closed traverse} = \frac{1}{2}\left(\sum P - \sum Q\right)$$

$$= \frac{1}{2}(282066.65-153362.66)$$

$$= \mathbf{64351.995}\ m^2$$

4. Compute the area of the traverse whose details are as under:

Line	Length in m	Bearing
AB	204	$87^0 30'$
BC	226	$20^0 20'$
CD	187	$280^0 00'$
DE	192	$210^0 30'$
EA	87.85	$180^0 28'$

Solution:

The consecutive coordinates are arranged in independent coordinate form as follows:

The independent coordinates of A are assumed to be (+200,+100)

Station	Side	Consecutive Coordinates		Independent coordinates	
		Latitude $l\cos\theta$	Departure $l\sin\theta$	North (y)	East (x)
A				200.0	100.0
B	AB	8.89	203.80	200+8.89 = 208.89	100+203.80 =303.80
C	BC	211.91	78.53	208.89+211.91=420.8	303.80+78.53 =382.33
D	CD	32.47	-184.15	420.8+32.47 =453.27	382.33-184.15 =198.18
E	DE	-165.43	-97.45	453.27-165.43=287.84	198.18-97.45 =100.73
A(check)	EA	-87.84	-0.73	287.84-87.84 =200.0	100.73-0.73 =100.0

Thus , the independent coordinates of all the stations become positive (i.e., they come to the first quadrant).

The coordinates are now arranged in determinant form as follows:

y_1	y_2	y_3	y_4	y_5	y_1
x_1	x_2	x_3	x_4	x_5	x_1
200	208.89	420.8	453.27	287.84	200
100	303.8	382.33	198.18	100.73	100

Sum of the product of co-ordinates indicated by thick lines.

i.e., $\sum P = y_1x_2 + y_2x_3 + y_3x_4 + y_4x_5 + y_5x_1$

$$\sum P = 200\times303.8 + 208.89\times382.33 + 420.8\times198.18 + 453.27\times100.73 + 287.84\times100 = 298460.94$$

Sum of the product of co-ordinates indicated by dotted lines

ie., $\sum Q = x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 + x_5y_1$

$$\sum Q = 100\times 208.89 + 303.8\times 420.8 + 382.33\times 453.27 + 198.18\times 287.84 + 100.73\times 200 = 399216.89$$

Therefore, Area of the traverse, $A = \frac{\sum P - \sum Q}{2} = \frac{298460.94 - 399216.89}{2} = 50377.98m^2$

5. The following are the latitudes and departures of the lines of a closed traverse ABCDA.

Line	Latitude(m)	Departure(m)
AB	-116.1	-44.4
BC	+6.8	+58.2
CD	80.5	17.2
DA	28.8	-31.0

Compute ate area of the traverse by Co-ordinate method.

Solution:

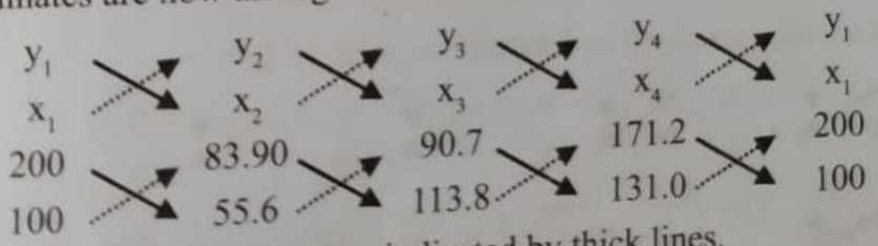
The consecutive coordinates are arranged in independent coordinate form as follows:

The independent coordinates of A are assumed to be (+200,+100)

Station	Side	Consecutive coordinates		Independent coordinates	
		Latitude	Departure	North (y)	East (x)
A				200.0	100.0
B	AB	-116.1	-44.4	+200.0-116.1 = 83.90	+100.0-44.4 =55.6
C	BC	+6.8	+58.2	+83.9+6.8 = 90.7	+55.6+58.2 =113.8
D	CD	80.5	17.2	+90.7+80.5 =171.2	+113.8+17.2 =131.0
A (check)	DA	28.8	-31.0	+171.2+28.8 =200.0	+131.0-31.0 =100.0

Thus, the independent coordinates of all the stations become positive (i.e., they come to the first quadrant).

The coordinates are now arranged in determinant form as follows:



Sum of the product of co-ordinates indicated by thick lines.

i.e., $\sum P = y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1$

$$\sum P = 200\times 55.6 + 83.9\times 113.8 + 90.7\times 131.0 + 171.2\times 100 = 49669.52$$

Sum of the product of co-ordinates indicated by dotted lines

i.e., $\sum Q = x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1$

$$\sum Q = 100\times83.9+55.6\times90.7+113.8\times171.2+131\times200 = 59115.48$$

Therefore, Area of the traverse, $A = \dfrac{\sum P - \sum Q}{2} = \dfrac{49669.52-59115.48}{2} = 4722.98m^2$

6. Find the area of the closed traverse having the following data by the co-ordinate method.

Line	Latitude(m)	Departure(m)
AB	225.5	120.5
BC	-245	210
CD	-150.5	-110.5
DA	170	-220

Assume the independent co-ordinate of point A as X-100, Y-200.

Solution:

The consecutive coordinates are arranged in independent coordinate form as follows:

The independent coordinates of A are assumed to be (+200, +100)

Station	Side	Consecutive coordinates		Independent coordinates	
		Latitude	Departure	North (y)	East (x)
A				200.0	100.0
B	AB	225.5	120.5	+200.0+225.5=425.5	+100.0+120.5 =220.5
C	BC	-245	210	+425.5-245 =180.5	+220.5+210 = 430.5
D	CD	-150.5	-110.5	+180-150.5 = 30	+430.5-110.5 = 320
A(check)	DA	170	-220	+30+170 =200.0	+320-220 =100.0

Thus, the independent coordinates of all the stations become positive (i.e., they come to the first quadrant).

The coordinates are now arranged in determinant form as follows:

$$\begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_1 \\ x_1 & x_2 & x_3 & x_4 & x_1 \\ 200 & 425.5 & 180.5 & 30 & 200 \\ 100 & 220.5 & 430.5 & 320 & 100 \end{array}$$

Sum of the product of co-ordinates indicated by thick lines.

i.e., $\sum P = y_1x_2 + y_2x_3 + y_3x_4 + y_4x_5 + y_5x_1$

$$\sum P = 200\times220.5+425.5\times430.5+180.5\times320+30\times100 = 288037.75$$

$\Sigma Q = x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 + x_5y_1$

$\Sigma Q = (100\times425.5)+(220.5\times180.5)+(430.5\times30)+(320\times200)$

$= 159265.25$ $\therefore$ Area $= \dfrac{\Sigma P - \Sigma Q}{2} = 64386.25\ m^2$

1.30.5 OMITTED MEASUREMENTS

1. **Following are the lengths and bearings of traverse PQRS:**

Line	Length in m	Bearings
PQ	248.0	30^0
QR	320.0	140^0
RS	180.0	210^0

Calculate the length and bearing of SP.

Solution:

Let l be the length and θ be the bearing of line SP. Then,

$$\sum Latitude = 0$$

$$248\cos 30^0 + 320\cos 140^0 + 180\cos 210^0 + l\cos\theta = 0$$

$$\therefore l\cos\theta = 186.24\,\text{m}$$

$$\sum Departure = 0$$

$$248\sin 30^0 + 320\sin 140^0 + 180\sin 210^0 + l\sin\theta = 0$$

$$\therefore l\sin\theta = -239.69\,\text{m}$$

Latitude +ve and departure –ve. Hence , the bearing is in 4th quadrant.

$$l = \sqrt{186.24^2 + 239.69^2} = 303.54m$$

$$\tan\theta = \frac{239.69}{186.24}$$

$$\theta = 52^0 9'10'' = N52^0 9'10''W$$

$$\theta = 360^0 - 52^0 9'10'' = 307^0 50'56''$$

2. **From the following data of closed traverse, calculate the length and bearing of the line SP.**

Line	Length in m	Bearing
PQ	85	N83°36'E
QR	137	N42°15'E
RS	67	N63°48'W
SP	?	?

Solution:

Let l be the length and θ be the bearing of line SP. Then,

$\sum Latitude = 0$

$85\cos 83^0 36' + 137\cos 42^0 15' + 67\cos 63^0 48' + l\cos\theta = 0$

$140.46 + l\cos\theta = 0$

$\therefore l\cos\theta = -140.46$

$\sum Departure = 0$

$85\sin 83^0 36' + 137\sin 42^0 15' - 67\sin 63^0 48' + l\sin\theta = 0$

$116.47 + l\sin\theta = 0$

$\therefore l\sin\theta = -116.47$

Both latitude and departure are –ve. Hence , the bearing is in 3rd quadrant.

$l = \sqrt{140.46^2 + 116.47^2} = 182.46m$

$\tan\theta = \dfrac{116.47}{140.46} = 0.82$

$\theta = 39^0 39'57'' = S39^0 39'57''W$

$\theta = 180^0 + 39^0 39'57'' = 219^0 39'57''$

3. The following are the lengths and bearings of the sides of closed traverse ABCD.

Line	Length in m	Bearings
AB	78.2	$140^0 12'$
BC	198.0	$36^0 24'$
CD	37.8	$338^0 48'$
DA	?	?

Calculate the length and bearing of DA.

Solution:

Let l be the length and θ the bearing of line DA. Then

$\sum L = 0$

$78.2\cos 140^0 12' + 198.0\cos 36^0 24' + 37.8\cos 338^0 48' + l\cos\theta = 0$

$134.53 + l\cos\theta = 0$

$\therefore l\cos\theta = -134.531$

$\sum D = 0$

$78.2\sin 140^0 12' + 198.0\sin 36^0 24' + 37.8\sin 338^0 48' + l\sin\theta = 0$

$153.884 + l\sin\theta = 0$

$\therefore l\sin\theta = -153.884$

Both latitude and departure are –ve. Hence, the bearing is in 3rd quadrant.

$l = \sqrt{134.531^2 + 153.884^2} = 204.399m$

$\tan\theta = \frac{153.884}{134.531} = 1.143$

$\theta = 48^0 50'20'' = S48^0 50'20''W$

$\theta = 180^0 + 48^0 50'20'' = 228^0 50'20''$

4. The following length and bearings were recorded in running a Traverse ABCDE. The length and bearing of EA have been omitted. Calculate the length and bearing of EA.

Line	Length in m	Bearings
AB	217.5	$120^0 15'$
BC	318.0	$62^0 30'$
CD	375.0	$322^0 24'$
DE	283.5	$335^0 18'$
EA	?	?

Solution:

Let l be the length and θ be the bearing of line EA. Then,

$\sum Latitude = 0$

$217.5\cos 120^0 15' + 318\cos 62^0 30' + 375\cos 322^0 24' + 283.5\cos 335^0 18' + l\cos\theta = 0$

$591.62 + l\cos\theta = 0$

$\therefore l\cos\theta = -591.62$

$\sum Departure = 0$

$217.5\sin 120^0 15' + 318\sin 62^0 30' + 375\sin 322^0 24' + 283.5\sin 335^0 18' + l\sin\theta = 0$

$122.68 + l\sin\theta = 0$

$\therefore l\sin\theta = -122.68$

Both latitude and departure are –ve. Hence, the bearing is in 3rd quadrant.

$l = \sqrt{591.62^2 + 122.68^2} = 604.49m$

Length of EA $= 604.49m$

$\tan\theta = \frac{122.68}{591.62} = 0.207$

$\theta = 11^0 42'54'' = S11^0 42'54''W$

$\theta = 180^0 + 11^0 42'54'' = 191^0 42'54''$

5. **During a theodolite survey, following details are noted:**

Line	Length in m	Bearings
AB	550	60^0
BC	1200	115^0
CD	?	?
DA	1050	310^0

Calculate the length and bearing of the line CD.

Solution:

Let l be the length and θ be the bearing of line CD. Then,

$$\sum Latitude = 0$$

$$550\cos 60^0 + 1200\cos 115^0 + l\cos\theta + 1050\cos 310^0 = 0$$

$$442.78 + l\cos\theta = 0$$

$$\therefore l\cos\theta = -442.78$$

$$\sum Departure = 0$$

$$550\sin 60^0 + 1200\sin 115^0 + l\sin\theta + 1050\sin 310^0 = 0$$

$$759.53 + l\sin\theta$$

$$\therefore l\sin\theta = -759.53$$

Both latitude and departure are –ve. Hence, the bearing is in 3rd quadrant.

$$l = \sqrt{442.78^2 + 759.53^2} = 879.17m$$

$$\tan\theta = \frac{759.53}{442.78} = 1.71$$

$$\theta = 59^0 45'33'' = S59^0 45'33''W$$

$$\theta = 180^0 + 59^0 45'33'' = 239^0 45'33''$$

6. **The following data was recorded in running a traverse in counter clockwise direction, the length CD and bearing DE have been omitted.**

Line	Length in m	Bearing
AB	281.4	$110^0 49'$
BC	129.4	$21^0 49'$
CD	?	$340^0 26'$
DE	144.5	?
EA	168.7	$254^0 24'$

Determine the omitted quantities.

7. **The following data were recorded in running a traverse, the length of AB and CD have been omitted:**

Line	Length in m	Bearing
AB	?	$33^0 45'$
BC	300	$86^0 23'$
CD	?	$169^0 23'$
DE	450	$243^0 54'$
EA	268	$317^0 30'$

Determine the omitted quantities.

Solution:

Let length of AB be the l_1 and that of CD be the l_2.

$$\sum Latitude = 0$$

$$l_1 \cos 33^0 45' + 300 \cos 86^0 23' + l_2 \cos 169^0 23' + 450 \cos 243^0 54' + 268 \cos 317^0 30' = 0$$

$$0.83 l_1' + 18.92 - 0.98 l_2 - 197.97 + 197.59' = 0$$

$$0.83 l_1' - 0.98 l_2 = -18.54 ------1$$

$\sum Departure = 0$

$l_1 \sin 33^0 45' + 300 \sin 86^0 23' + l_2 \sin 169^0 23' + 450 \sin 243^0 54' + 268 \sin 317^0 30' = 0$

$0.56 l_1 + 299.40 + 0.18 l_2 - 404.11 - 181.06' = 0$

$0.56 l_1 + 0.18 l_2 = 285.77 ------2$

From equation 1

$0.83 l_1 = 0.98 l_2 - 18.54$

$l_1 = \frac{0.98 l_2}{0.83} - \frac{18.54}{0.83} = 1.18 l_2 - 22.34$

Substituting it in equation 2

$0.56(1.18 l_2 - 22.34) + 0.18 l_2 = 285.77$

$0.66 l_2 - 12.51 + 0.18 l_2 = 285.77$

$0.66 l_2 + 0.18 l_2 = 285.77 + 12.51 = 298.28$

$0.84 l_2 = 298.28$

$l_2 = 355m$

$l_1 = 1.18 \times 355 - 22.34 = 396.56m$

8. Following are details of closed traverse. Find the missing quantities.

Line	Length in m	Bearing
AB	298.7	0^0
BC	205.7	$N25^0 12'W$
CD	?	$S75^0 6'W$
DE	?	$S56^0 24'W$
EA	213.4	$N35^0 36'E$

Solution:

Let length of CD be the l_1 and that of DE be the l_2. Then,

$\sum Latitude = 0$

$298.7 \cos 0^0 + 205.7 \cos 25^0 12' - l_1 \cos 75^0 6' - l_2 \cos 56^0 24' + 213.4 \cos 35^0 36' = 0$

$298.7 + 186.12 - 0.26 l_1 - 0.55 l_2 + 173.51 = 0$

$-0.26 l_1' - 0.55 l_2 = -658.33$

$0.26 l_1 + 0.55 l_2 = 658.33 ------1$

l_3 14

$\sum Departure = 0$ l_3 l_4

$298.7\sin 0^0 - 205.7\sin 25^0 12' - l_1 \sin 75^0 6' - l_2 \sin 56^0 24' + 213.4 \sin 35^0 36' = 0$

$0 - 87.58 - 0.96l_1 - 0.81l_2 + 124.22 = 0$

$-0.96l_1' - 0.81l_2 = -36.64$

$0.96l_1 + 0.81l_2 = 36.64 - - - - - - -2$

From equation 1

$0.26l_1 = 658.33 - 0.55l_2$

l_3 $l_1 = \frac{658.33}{0.26} - \frac{0.55l_2}{0.26} = 2532.03 - 2.11l_2$

Substituting it in equation 2

$0.96(2532.03 - 2.11l_2) + 0.81l_2 = 36.64$

$2430.74 - 2.03l_2 + 0.81l_2 = 36.64$

$-1.22l_2 = 36.64 - 2430.74 = -2394.1$

l_4 $l_2 = \frac{-2394.1}{-1.22} = 1962.37m$

l_3 $\therefore l_1 = 2532.03 - 2.11 \times 1962.37 = 1608.57m$

... is to be set out at 90⁰ to the given line AB. This is

11. The following notes refer to part of a traverse survey.

Line	Length in m	Bearing
PQ	186.00	$30^0 15'$
QR	649.00	$134^0 32'$
RS	424.00	$222^0 18'$

Compute the distance between point A on PQ 112.0 m from P and point B on RS 288.0 m from R.

Solution:

As the A and B are on PQ and RS, the bearing of AQ and RB are same as those of PQ and RS respectively.AQRB is closed traverse.

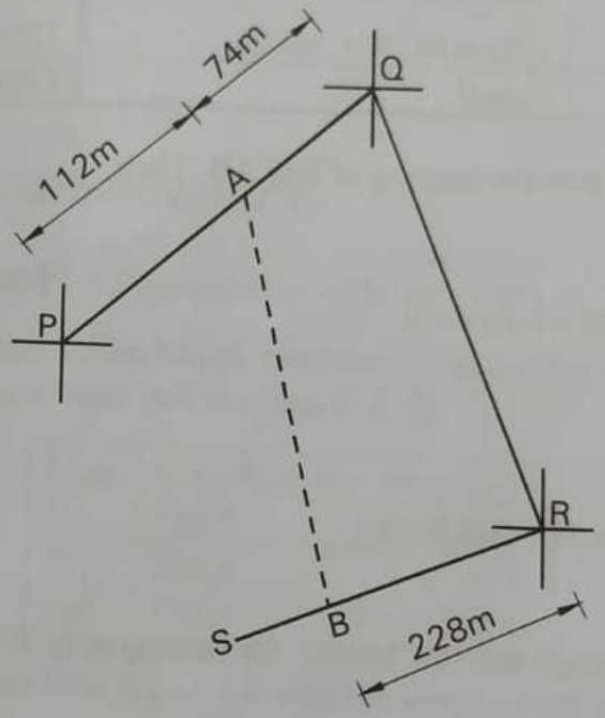


Fig. 1.28

Calculating latitude and departure :

Line	Length	Bearing (θ)	Latitude	Departure
AQ	186-112=74m	$30^0 15'$	$74\cos 30^0 15' = 63.96$	$74\sin 30^0 15' = 37.27$
QR	649m	$134^0 32'$	$649\cos 134^0 32' = -455.6$	$649\sin 134^0 32' = 462.63$
RB	288m	$222^0 18'$	$288\cos 222^0 18' = -213.0$	$288\sin 222^0 18' = -193.82$
BA	$l = ?$		$l\cos\theta$	$l\sin\theta$

Let l be the length and θ be the bearing of line BA. Then,

$$\sum L = 0$$

$$63.96 - 455.16 - 213 + l\cos\theta = 0$$

$$\therefore l\cos\theta = 604.20$$

$$\sum D = 0$$

$$37.27 + 462.63 - 193.82 + l\sin\theta = 0$$

$$\therefore l\sin\theta = -306.08$$

$$l = \sqrt{604.20^2 + 306.08^2} = 677.30m$$

$\therefore$ Distance AB = 677.30 m

$$\tan\theta = \frac{306.08}{604.20} = 0.507$$

$$\theta = 26^0 52'50'' = N26^0 52'50''W$$

$$\theta = 360^0 - 26^0 52'50'' = 333^0 7'10''$$

12. In the traverse ABCDA, the following observations were made

F.B of DA

13. **The following details are given for a traverse survey where length of AB is required. Point A is 50m from point P and at B is 75m from point R.**

Line	Length in m	Bearing
PQ	125.50	N30°15'E
QR	80.25	S40°30'E
RS	150.75	S60°30'W

Solution:

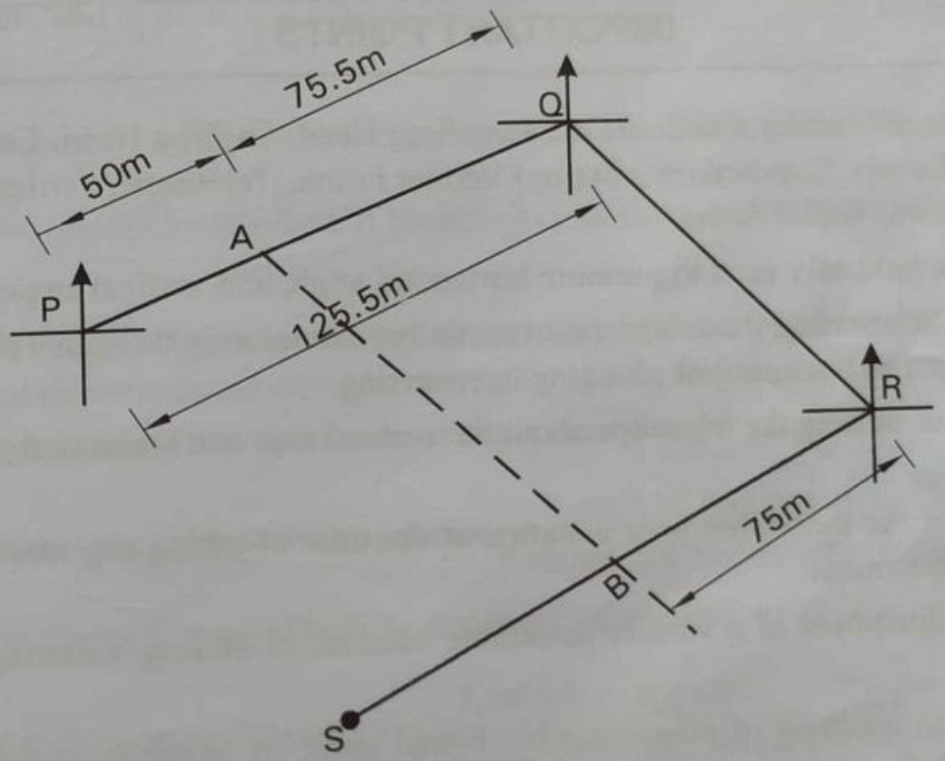


Fig. 1.30

AQRB is closed traverse.

$$\sum L = 0$$

$$75.5\cos 30^0 15' - 80.25\cos 40^0 30' - 75\cos 60^0 30' + lat_{BA} = 0$$

$$65.21 - 61.02 - 36.93 + lat_{BA} = 0$$

$$lat_{BA} = 32.74m$$

$$\sum D = 0$$

$$75.5\sin 30^0 15' + 80.25\sin 40^0 30' - 75\sin 60^0 30' + dep_{BA} = 0$$

$$38.03 + 52.11 - 65.27 + dep_{BA}$$

$$dep_{BA} = -24.87m$$

$$\text{Length of BA} = \sqrt{Latitude^2 + Departure^2} = \sqrt{32.74^2 + 24.87^2} = 41.11m$$

$$\tan\theta = \frac{Departure}{Latitude} = \frac{24.87}{32.74} = 0.76$$

$$\theta = 37^0 13'16'' = N37^0 13'16''W$$

$$W.C.B = 360^0 - 37^0 13'16'' = 322^0 46'44''$$

IMPORTANT POINTS

1. Various parts of vernier theodolite are Levelling Head, Shifting Head, Lower Plate, Upper Plate, Plate Levels, Standard (or a Frame), Vernier Frame, Telescope, Vertical Circle, Altitude Bubble, Screws, Tripod Stand.
2. Theodolite is basically used to measure horizontal angle and vertical angle.
3. The process of rotating the telescope over the horizontal axis through 180° in the vertical plane is known as transiting or plunging or reversing.
4. The process of turning the telescope about the vertical axis in a horizontal plane is known as swinging.
5. The setting of the theodolite over a station at the time of taking any observation is called temporary adjustment.
6. Temporary adjustment of a vernier theodolite consists of Setting, Centring, Levelling and Focussing.
7. Most Common methods of measuring horizontal angle by using theodolite are Repetition method and Reiteration method.
8. Errors that are eliminated by repetition method are errors due to eccentricity of verniers and centres, imperfect adjustment of instrument, inaccurate graduations and inaccurate bisection of the signal.
9. Sources of errors in theodolite surveying are Instrumental errors, Personal errors, Natural errors and Mistakes.
10. A traverse consists of a series of straight lines connected successively at established points along the route of a survey.
11. Steps involved in a traverse survey with theodolite are Reconnaissance, Selection of stations

5. Deflection is the angle between forward line of a traverse and prolongation of preceding line.
6. Process of turning the telescope in horizontal plane is called transiting.
7. The method of reiteration of measuring horizontal angles is generally preferred when several angles are to be measured at a particular station.
8. The line of collimation must be parallel to the horizontal axis.
9. The distance by which the last point of the traverse falls short to coincide with the starting points is called the closing error.

VI Match the list type questions

i) Match the list I (Operation) and List II (Purpose)

	List I (Operation)		List II (Purpose)	
A	Centering theodolite	1	Turning telescope in vertical plane	B
B	Transiting theodolite	2	Moving telescope about vertical axis	C
C	Swinging theodolite	3	Keeping vertical circle to the left	D
D	Telescope normal	4	Brining vertical axis over the ground	A

ii) Match the list I (Errors) and List II (Remedial measures)

	List I (Errors)		List II (Remedial measures)	
A	Error due to eccentricity of verniers	1	By focussing the eye piece and objective properly	D
B	Error due to non-adjustment of line of sight	2	By taking both face observations	
C	Error due to non-uniform graduations	3	By reading both verniers and taking the mean of two	A
D	Error due to parallax of telescope	4	By taking mean of the several readings distributed over different portions of the graduated circle.	C

VII Theodolite Surveying and Traversing Quiz . Multiple choice - Questions.

1. Theodolite is an instrument used for measuring
   (a) Horizontal angles (b) Vertical angles
   (c) Both horizontal and vertical angles (d) distances
2. The process of turning the telescope about the horizontal axis in a vertical plane by 180° revolution is known as
   (a) Reversing (b) Transiting (c) Plunging (d) All of the above